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Abstract of the Dissertation

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Professor Christopher S. Tang, Chair

This dissertation consists of two chapters. The first is at the socially responsible operations in the agricultural sector of emerging economies. The agricultural sector in emerging markets accounts for a significant portion of economic activities even though most farmers are trapped in the poverty cycle owing to their smallholdings. Aggregating these farmers through formal or informal cooperatives (coops) can enable them to: (1) reduce production cost; (2) increase/stabilize process yield; (3) increase brand awareness; (4) eliminate unnecessary intermediaries; and (5) eliminate price uncertainty. To examine whether these effects will benefit the members of such aggregation when they compete with other individual farmers, we present separate models to capture the essence of these five effects. For each effect, we find that it is beneficial for a farmer to be part of the aggregation only when the size of the aggregation is below a certain threshold. Also, while certain effects are beneficial to the market as a whole, other effects are hurtful due to higher market price and/or lower production quantity.

The second chapter examines the price matching policy in the ocean freight industry. Ocean freight continues to play the most significant role in supporting material flows along global supply chains. In this industry, shippers (customers) can purchase freight services either directly from a carrier (service provider) in advance or through a freight forwarder

(spot market) just before the departure of an ocean liner. To entice shippers to book directly from the carrier, we develop a new variant of basic price matching policy, so called "fractional price matching" where the carrier refund only a "fraction" of the price difference. By modeling the dynamics between the carrier and the shippers as a Stackelberg game, we show that the carrier can use the fractional price matching contract to generate a higher demand from the shippers by increasing the fraction in equilibrium. Also, there are multiple optimal regular prices and the optimal fraction that possess the following property: if the carrier increases the fraction, then the carrier should increase the regular price to compensate for bearing additional risk. More importantly, we find that the optimal fractional price matching contract is revenue neutral in the sense that it enables the carrier to obtain the same expected revenue as before (when there was no price matching). This result implies that the carrier can develop a menu of fractional price matching contracts that are revenue neutral, and let the shipper to choose a specific contract as desired.

The dissertation of Jaehyung An is approved.

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2013

To my family

Table of Contents

1 Introduction and Overview **1**

2 Aggregating Smallholder Farmers for Alleviating Poverty in Emerging Economies **4**

2.1 Introduction 4

2.2 Literature Review 8

2.3 Focusing on cost reduction 10

2.4 Focusing on process yield improvement 13

2.5 Focusing on improving brand awareness 17

2.6 Focusing on improving selling prices using direct channel 20

2.7 Focusing on reducing selling price risks 24

2.8 Concluding Remarks 27

3 Fractional Price Matching Policies Arising from the Ocean Freight Service Industry **33**

3.1 Introduction 33

3.2 Literature Review 37

3.3 Fractional Price Matching Policies 40

3.3.1 The carrier's pricing behavior and the shipper's booking behavior . . 42

3.3.2 The carrier's problem 43

3.4 Optimal Fractional Price Matching 44

3.4.1 Optimal Fractional Price Matching: when the regular price r is given 46

3.4.2 Numerical results for the case when the spot price s is Normally distributed . . . 47
3.5 Extension 1: General Shipper Demand . . . 49
3.5.1 The carrier's problem . . . 50
3.6 Extension 2: Dependent Spot Price . . . 51
3.6.1 Special case: when $\beta = 0$. . . . 52
3.6.2 General case: when $\beta \in [0, 1]$ . . . 53
3.7 Conclusion . . . 56
References . . . **64**

List of Figures

2.1 Unified Framework 32

2.2 Supply Chain Configuration 32

List of Tables

2.1 Equilibrium Outcomes 32

3.1 Impact of μ: Optimal Pricing Policy 60

3.2 Impact of σ: Optimal Policy 60

3.3 Impact of μ_ϵ: Optimal Pricing Policy (when $\tau = 1$) 61

3.4 Impact of σ_ϵ: Optimal Policy (when $\tau = 1$) 62

3.5 Impact of τ: Optimal Policy 63

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CHAPTER 1

Introduction and Overview

This dissertation consists of two chapters. Each of the subsequent two chapters investigates operations questions in diverse business environments. More specifically, Chapter 2 examines the socially sustainable operations in the agricultural sector of emerging economies, and Chapter 3 investigates the price matching policy in the ocean freight industry. Below I overview each chapter and summarize the implications.

Chapter 2 is motivated by growing interest in socially responsible operations in emerging economies. In emerging markets, the agricultural sector accounts for a significant portion of economic activities even though most farmers are trapped in the poverty cycle owing to their smallholdings. As a way to alleviate poverty among these farmers in emerging economies, government agencies, NGOs, social enterprises, and private companies are aggregating farmers to establish formal or informal cooperatives (coops) as well as self-help groups. Aggregating farmers through formal or informal cooperatives (coops) can enable them to: (1) reduce production cost; (2) increase/stabilize process yield; (3) increase brand awareness; (4) eliminate unnecessary intermediaries; and (5) eliminate price uncertainty. To examine whether these effects will benefit the members of such aggregation when they compete with other individual farmers, we present separate models to capture the essence of these five effects.

Our analysis shows that the five effects can have different impacts on the market as a whole and on the equilibrium size of the aggregation under open or exclusive membership. In particular, our analysis of the equilibrium size of the aggregation explains why not all farmers join the aggregation in practice even when each of the above effects is beneficial to aggregation members. This research contributes to the nascent literature on social enterprise or social business, whereby for-profit or not-for-profit organizations seek to aggregate micro-entrepreneurs (e.g., smallholder farmers) so as to help alleviate poverty. Even though we analyze separate models to isolate the impact of each effect, our models can serve as building blocks for analyzing the case when multiple effects are present simultaneously in a single aggregation.

In Chapter 3, we investigates the pricing issues in ocean freight industry. Even though ocean freight continues to play the most significant role in supporting material flows along global supply chains, very little literature has addressed sea logistics while air and land logistics have been studies extensively. More specifically, the pricing and contracting issues arising from the ocean transportation industry in a B2B environment are not well understood, unlike the commercial airline industry that has a well-established system to deal with issues such as dynamic pricing, over-booking, etc. There are two major players in the ocean freight industry, where shippers (customers) can purchase freight services either directly from a carrier (service provider) in advance or through a freight forwarder (spot market) just before the departure of an ocean liner. To entice shippers to book directly from the carrier, we examine a situation when a carrier offers a fractional price matching contract that can be described as follows: the shipper pays a regular freight price in advance. However, the shipper will get a refund only if the realized spot price is below the regular price, where the refund is based on a fraction of the difference between the regular price and the spot price.

We model the dynamics between the carrier and the shippers as a Stackelberg game in which the carrier acts as the leader who sets the fractional price matching policy, and the shippers are the followers who decide whether to book with the carrier in advance. By anticipating the booking behavior of the shippers, we determine the optimal fractional pricing matching policy for the carrier. Our analysis enables us to draw the following conclusions. First, we show that the carrier can use the fractional price matching contract to generate a higher demand from the shippers by increasing the fraction in equilibrium. Second, there are multiple optimal regular prices and the optimal fraction. However, the optimal fractional price matching contract exhibits the following property: if the carrier offers a higher fraction, then the carrier should increase the regular price to compensate for bearing additional risk. Third, we find that the optimal fractional price matching contract is revenue neutral in the sense that it enables the carrier to obtain the same expected revenue as before (when there was no price matching). This result implies that the carrier can develop a menu of fractional price matching contracts that are revenue neutral and let the shipper to choose a specific contract as desired. Finally, we show that our results continue to hold when certain assumptions are relaxed.

CHAPTER 2

Aggregating Smallholder Farmers for Alleviating Poverty in Emerging Economies

2.1 Introduction

In emerging markets, the agricultural sector accounts for a significant portion of economic activities. For example, in India, this sector accounts for 50% of the workforce and for 17% of its GDP. Despite fast economic growth, most farmers in India and other emerging economies such as Brazil or China remain poor partly due to the fact that most farmers own less than 2 hectares of land. According to the World Bank, over 660 million people in rural areas earned less than US$2 per day in 2011. As stated in the "*India Country Overview* 2008" report issued by the World Bank (http://www.worldbank.org.in), there is a concern over current agricultural practices that are neither economically nor environmentally sustainable. Specifically, India's yields for many agricultural commodities are low, partly because farmers have limited access to advanced farming practice, reliable market information, and efficient sales channels, not to mention other obstacles such as access to credits, loans, water and electricity.[1]

As a way to alleviate poverty among farmers in emerging economies, government agencies, NGOs, social enterprises, and private companies are aggregating farmers to establish formal or informal cooperatives (coops) as well as self-help groups. For example, in India, the Na-

[1] Recently, [DGM12] examine a water distribution problem arising from inefficient surface water allocations among farmers that yield sub-optimal productivity. To overcome this inefficiency that is caused by the natural flow of water that goes through primary farms first and secondary farms second, they propose different mechanisms (internal payment, water guarantee, and reward) that help both primary and secondary farms to achieve socially-optimal water allocation.

tional Co-operative Development Corporation (NCDC), established in 1963, is a statutory Corporation under the Ministry of Agriculture. NCDC helps (formal or informal) cooperatives to create common infrastructural facilities and other income-generating assets to alleviate poverty in rural areas. Amul (www.amul.com), the largest food brand in India and the world's largest pouched milk brand with annual revenue of US $2.2 billion, is a cooperative. Private initiatives such as those of ITC's e-Choupal in India (www.itcportal.com) and Walmart's Direct Farm program in China (http://www.wal-martchina.com) are the other examples of aggregating smallholder farmers. Self-help groups are other forms of aggregation; e.g., the Sadhu Chaithanya Self Help Group in Kerala (India) conducts training for illiterate women from the farming communities in vegetable cultivation, mushroom cultivation, fisheries, plant propagation, bee keeping, etc. so that these women can improve their earnings (http://www.hindu.com/seta/2009/03/05/stories/2009030550161500.htm).

A notable aggregation takes the form of a formal cooperative. The modern cooperative began in Europe in the late 19th century as a mechanism to help farmers to alleviate poverty ([Hoy89]). An agricultural cooperative is an association that is managed by its farmer members who cooperate with other coop members to generate economic benefit for its members. According to the national council of farmer cooperatives (www.ncfc.org), there are over 3000 cooperatives with over 2 million farmers in the United States.

Aggregations of farmers focus on providing different benefits to their members, including: (1) helping members to reduce their cost by consolidating their purchase of seeds, fertilizers, and farm machinery; and (2) helping members to market their crops by creating brand awareness. While aggregations of farmers can potentially provide these benefits, the net effect is unclear because "aggregation members" (i.e., farmers who are part of the aggregation) and other "individual farmers" (i.e., farmers who are not part of the aggregation) do compete in the commodity market. For example, while aggregation members enjoy lower production costs, their selling price may become lower if competition from "other individual farmers" drives them to produce more. Consequently, their profit may end up lower even though their production costs are lower. This observation has motivated us to develop analytical models to examine the impact of an aggregation of farmers on the production quantities of its members and other individual farmers, as well as its impact on the market in terms of selling price and total production quantity.

In Chapter 2, we consider a situation in which multiple farmers produce a common crop (e.g., corn, wheat, etc.). Because the farmers produce the same crop, we shall assume that they engage in Cournot competition (i.e., the farmers compete on outputs) so that the realized unit price for the farmer's crop is a decreasing function of the total amount produced by all farmers. We shall examine a situation in which there is one aggregation (which may be organized as an informal self-help group or a formal cooperative organized by a private company, a government agency or an NGO), and the membership of the aggregation is either "open" (under which each farmer is free to join) or "exclusive" (under which the admission of a new member is subject to the consent of existing members of the aggregation). The aggregation can create five different effects to alleviate poverty (Refer to Figure 2.1):

1. *Lower cost.* Each member of the aggregation can reduce his marginal production cost via: 1) aggregate purchase of raw material; 2) aggregate process by sharing facilities; or 3) aggregate investment in farming equipment and learning new farming techniques.

2. *Stable process yield.* The aggregation can facilitate mutual learning and idea exchanges for its members to reduce the uncertainty of their output quantities.

3. *Stronger brand.* By selling products under one brand, aggregation members can collaborate on their marketing and develop better brand awareness so as to increase the demand for their products.

4. *Shorter supply chain.* The aggregation can increase their selling price by eliminating channel inefficiency (e.g., by selling through a direct channel to the market).

5. *Exclusive direct sales with a guaranteed selling price.* When the market price is uncertain, the aggregation can utilize its scale to create an option by selling directly to a

firm that offers a guaranteed selling price (ex-ante). However, individual farmers who are not part of the aggregation can only sell to the market at uncertain price.

We examine each of these five effects by analyzing separate models so as to isolate the impact of each effect. Our analysis demonstrates the following results for each of the aforementioned effects:

1. *Lower cost.* The aggregation is beneficial to the market by reducing price and increasing total quantity. Under open membership, all farmers will join the aggregation. However, under exclusive membership, not all farmers will join the aggregation.

2. *Stable process yield.* As the uncertainty of the total output of the aggregation is reduced, the aggregation is beneficial to the market by reducing price and increasing total quantity. Under open membership, all farmers will join the aggregation. However, under exclusive membership, not all farmers will join the aggregation.

3. *Stronger brand.* The aggregation is beneficial to the market by increasing total quantity. However, improved brand awareness of the aggregation enables the aggregation to charge a higher price than other individual farmers. Under open membership, all farmers will join the aggregation. However, under exclusive membership, not all farmers will join the aggregation.

4. *Shorter supply chain.* The aggregation does not affect the market in terms of its price and total quantity. Under open membership, not all farmers will join the aggregation. However, under exclusive membership, once a farmer joined a partnership with downstream partner(s) to shorten his supply chain, he will block all potential farmers joining this partnership.

5. *Exclusive direct sales with a guaranteed selling price.* The price of the aggregation members is higher than the expected price of other individual farmers only when the demand volatility is sufficiently small. Under either open or exclusive membership, not all farmers will join the aggregation.

As shown above, the five effects can have different impacts on the market as a whole and on the equilibrium size of the aggregation under open or exclusive membership. In particular, our analysis of the equilibrium size of the aggregation explains why not all farmers join the aggregation in practice even when each of the above effects is beneficial to aggregation members. This research contributes to the nascent literature on social enterprise or social business, whereby for-profit or not-for-profit organizations seek to aggregate micro-entrepreneurs (e.g., smallholder farmers) so as to help alleviate poverty. Even though we analyze separate models to isolate the impact of each effect, our models can serve as building blocks for analyzing the case when multiple effects are present simultaneously in a single aggregation. However, we defer the analysis of various 'combined effects' to future research.

The rest of chapter is organized as follows. We provide a brief literature review in Section 2.2. Then from Section 2.3 to Section 2.7, we provide a number of practical examples of different aggregations with specific focus, and we present five different models to examine the impact of the five aforementioned operational effects on the following factors: (1) the equilibrium outcomes (production quantity and profit) associated with the aggregation members and other individual farmers; (2) the market price and the total output in equilibrium; (3) the size of the aggregation in equilibrium. The conclusion is provided in in Section 2.8.

2.2 Literature Review

Our research seeks to contribute to the recent literature on social enterprises. Underlying our study is the research stream in economics that investigates cooperative behavior in a competitive market especially in the context of multiple firms engage in joint research projects. For example, [DJ88], [Cho93], and [KMZ92] generally assume that, while firms engage in Cournot competition, they also cooperate in conducting joint R&D projects for cost reduction. [DJ88] and [Cho93] assume multi-stage decision mechanisms in duopolies where R&D competition is followed by a product market competition under Cournot assumptions. [KMZ92] extend this model to the oligopoly case. Further, there are studies investigating the equilibrium membership size of research consortia (e.g., [Kat86], [Poy95], [DW97]), which are analogous to our analysis of finding the size of the aggregation in equilibrium. While our objectives are similar to this stream of research, our context is different. Specifically, we consider five different operational effects of aggregating smallholder farmers, and we provide different managerial insights about aggregating smallholder farmers for alleviating poverty in emerging economies.

Because the five different effects of an aggregation stated in Section 2.1 have operational impacts on the production of farmers, our research is related to the literature in operations management which specifically investigates firms' strategic production decisions with cooperation in a competitive market. [SR86] investigate cooperative behavior among oligopolistic firms that supply a homogeneous product. By using cooperative game theory, they use several alternative characteristic functions under Shapley value to examine whether the firm would choose to remain separate or form a grand coalition (i.e., all firms join the coop). When each producer faces a linear production problem, [Gra86] studies a cooperative game where producers can share their resources, and centralize their production. By extending the model examined by [Gra86], [FFG05] analyze a cooperative game where producers can centralize their production, but cannot share their resources. While this stream of research focuses on the conditions under which *all farmers will form a grand coalition* using cooperative game theory, we employ a non-cooperative game model in order to analyze practical situations in which not all farmers join an aggregation so that aggregation members and other individual farmers compete (non-cooperatively) in the market. We show that *some but not all firms may join an aggregation* (i.e., partial coalition) in equilibrium even under open membership.

As interests in socially responsible business grow, various researchers have identified different dimensions of value creation in social enterprises ([LAS10], [ST11], [SBY12]). [SBY12] study how value gets created when social enterprises or for-profit companies collaborate with the poor (e.g., farmers). For example, [ST12] develop stylized models to examine various operational mechanisms (e.g., last mile delivery, info-mediation) of social enterprises analytically. [CSS12] investigate an India-based company, ITC which operates the "e-Choupals program" under which farmers can access valuable information such as price updating, weather forecast, and the best practice. They investigate ITC's incentive for providing such information. Besides offering information about the price of crops in local and wholesale markets through their e-Choupals program, ITC also offers farmers the option to sell their crops directly to ITC for a pre-specified price ([DAS11]). Although this stream of research has addressed different value-creation mechanisms of social enterprises in emerging economies, we analyze five other value-creation mechanisms through aggregation that are commonly used to create value for smallholder farmers. Because the impact of the effects of an aggregation (as stated in the Section 2.1) remains unclear especially when farmers are operating in a competitive environment, we are interested in examining the impact of these effects on the equilibrium outcomes of aggregation members, other individual farmers, and the market.

2.3 Focusing on cost reduction

Throughout Chapter 2, we consider a situation in which n symmetric (identical) smallholder farmers who produce a common crop (e.g., corn, wheat, etc.).[2] Because the farmers produce the same crop, we shall assume that they engage in Cournot competition (i.e., the farmers compete on outputs). Therefore, the realized unit price for the farmers' crop p can be expressed as $p = a - kQ$, where Q is the total amount produced by all n farmers. We examine a situation in which there is one aggregation (which may be organized as a self-help group or organized as a formal or informal coop by a government agency, an NGO, a social enterprise, or a private company),[3] and the aggregation membership is either "open" (under which each farmer is free to join) or "exclusive" (under which the admission of a

[2] Because we consider the case of poor and small farmers in developing countries, it is reasonable to assume that every farmer has the similar scale of operations.

[3] In practice, farmers decide to join an existing aggregation that has been initiated by a government or an NGO. Therefore, each farmer knows how many aggregations or groups exist in the market, and decides whether to join an aggregation or not. Hence in our model, we assume that the number of aggregations in the market is given exogenously. Specifically, we consider the case in which there exists only one aggregation with s members and $n-s$ individual farmers who are not part of the aggregation. This enables us to analyze the problem in closed-form solutions, and to allow us to compare equilibrium outcomes clearly.

new member is subject to the consent of existing aggregation members). Both types of membership admission mechanisms are commonly observed ([Blo01]). Figure 2.1 depicts our model setup as described.

In this section, we examine the case when the aggregation focuses on reducing the marginal production cost c. For example, consider the Chengalrayan coop sugar mills located in the Tamil Nadu of India. [Kar12] reports that this coop reduces its production cost by consolidating the purchase of fuel, oil, lubricants, and cane crushing equipment and by improving the production efficiency of the sugar mill operations (e.g., preventive maintenance to reduce downtime, and production planning and scheduling to increase output). In South Africa, agricultural coops have helped farmers to aggregate their purchase of seeds and fertilizers so that the coop members can reduce their supply cost ([OK07]).

Specifically, we consider the case when the unit production cost is $\frac{c}{s}$ for each of the s aggregation members.[4] For any farmer i who belongs to the aggregation with s (≥ 1) members and produces q_i units, his profit can be written as:[5]

$$\pi_i(s) = (p - \frac{c}{s})q_i = [(a - kQ) - \frac{c}{s}]q_i \ for\ i = 1, 2, ..., s. \tag{2.1}$$

Also, for any individual farmer j who is not part of the aggregation, his profit is:

$$\pi_j(s) = (p - c)q_j = [(a - kQ) - c]q_j \ for\ j = s + 1, s + 2, ..., n. \tag{2.2}$$

Throughout Chapter 2, we reserve the subscript i to denote an aggregation member and j to denote an individual farmer who is not part of the aggregation, respectively.

[4] We obtain similar structural results when the unit production cost is $[c_{min} + \frac{c}{s}]$, $[c - \delta(s - 1)]$ or $[c/\delta^s]$ where $\delta > 0$.

[5] When $s = 1$, there is only one member in the aggregation. Relative to other non-aggregation individual members, a single-member aggregation does not have any cost advantage in this setting. However, when we examine other effects (e.g., shortening the supply chain in Section 2.6), a single-member aggregation has an advantage over other non-aggregation members because this single-member aggregation can get a benefit from forming partnership with channel partners. Also, in many instances, there is an "admission fee" for a farmer joining an aggregation. This admission fee is intended to cover the upfront costs (e.g., initial investments, cost of financing, cost of coordination among aggregation members) for establishing an aggregation. To ease our exposition, we assume there is no admission fee for a farmer to join an aggregation. However, our model can be easily extended to the case when there is an admission fee. In Appendix 2.1, we illustrate how our model (for the cost reduction case) can be extended to the case when an admission fee is required for a farmer to join an aggregation.

By considering the first-order condition associated with (2.1) and (2.2), we establish the following result:

Proposition 1 *Suppose there is an aggregation with s members that focuses on cost reduction. Then for each aggregation member i, his output in equilibrium is:*

$$q_i^* = \frac{a + nc - (s-1)c}{k(n+1)} - \frac{c}{ks} \; for \; i = 1, 2, ..., s, \tag{2.3}$$

where q_i^* is increasing in s if and only if $s < \sqrt{n+1}$. For each individual farmer j, his output in equilibrium is:

$$q_j^* = \frac{a + nc - (s-1)c}{k(n+1)} - \frac{c}{k} \; for \; j = s+1, s+2, ..., n, \tag{2.4}$$

where q_j^* is decreasing in s. Moreover, $q_i^* - q_j^* = \frac{c}{k}(1 - \frac{1}{s}) \geq 0$ and $\pi_i(s) \geq \pi_j(s)$ in equilibrium for any given $s \geq 1$.

Proof All formal proofs are provided in Appendix 2.2.

Because each aggregation member enjoys a lower unit production cost, Proposition 1 suggests that each aggregation member i will always produce more than an individual farmer j in equilibrium.

Now, let us examine the impact of the aggregation on the market as a whole. By noting that the total output in equilibrium $Q^* = sq_i^* + (n-s)q_j^*$ and the market price in equilibrium $p^* = a - kQ^*$, it is easy to check from (2.3) and (2.4) that $Q^* = \frac{n(a-c)+(s-1)c}{k(n+1)}$ and $p^* = \frac{a+nc-(s-1)c}{n+1}$, where Q^* is increasing in s and p^* is decreasing in s. Therefore, we can conclude that, the market as a whole can benefit from the existence of an aggregation due to lower market price p^* and higher total quantity Q^*.

Finally, let us examine the size of the aggregation in equilibrium. As one would expect, Proposition 1 implies that $\pi_i(s) \geq \pi_j(s)$ so that an aggregation member i will always earn a higher profit than an individual farmer j. Consequently, if the admission is left to the farmers (i.e., under open membership), then all n farmers will join the aggregation so that the equilibrium aggregation size is equal to n. However, if the admission process requires the approval of existing aggregation members (i.e., under exclusive membership), then existing members would admit a new member only if this new member will improve the profit of the existing members. To determine the equilibrium size of the aggregation which maximizes the aggregation member's profit under exclusive membership, we can utilize Q^* and q_i^* given in Proposition 1 to check that the equilibrium size of an aggregation (i.e., aggregation size s that maximizes $\pi_i(s) = k(q_i^*)^2$) is $s^* = \sqrt{n+1}$. This result suggests that, under exclusive membership, the equilibrium aggregation size $s^* = \sqrt{n+1}$ $(< n)$. This result explains why various aggregations (such as coops) in practice do not include all farmers in their local communities.

2.4 Focusing on process yield improvement

When farmer's crop is subject to random yield, aggregations (or coops) can enable farmers to improve or stabilize their process yield. For example, the Agricultural Cooperatives for Ethiopia (ACE) program has fostered the development of cooperative farms in Ethiopia. Through the ACE program, coop members can learn the best farming skills to improve their process yield ([DA05]). Founded in 1984, the Kilimanjaro Native Coop Union (KNCU) is the Africa's oldest coop that serves 60,000 coffee farmer members who grow coffee on the slopes of Kilimanjaro (www.kncutanzania.com). The core function of this coop is to help farmers to improve their process yield of the highest quality coffee.

Traditionally, by processing q_j units, an individual farmer j's actual output is $o_j = z_j q_j$, where z_j is the process yield of farmer j that has $E(z_j) = \mu$ and $Var(z_j) = \sigma^2$. With the establishment of an aggregation with s members, an aggregation farmer i who processes q_i units will obtain an output, $o_i = y_i q_i$, where $E(y_i) = \mu_s$ and $Var(y_i) = \sigma_s^2$. Hence, the "total actual output" from all n farmers is equal to: $O = \sum_{i=1}^{s} y_i q_i + \sum_{j=s+1}^{n} z_j q_j$ (Here, we use O (instead of the deterministic variable Q) to denote the total quantity as a way to highlight the fact that O is a random variable). To capture the benefit of improved yield for aggregation members, we shall assume that $\mu_s \geq \mu$ and $\sigma_s^2 \leq \sigma^2$, where $\mu_s = \mu$ and $\sigma_s^2 = \sigma^2$

when $s = 1$. For ease of our exposition, we shall assume the process yields are independent; i.e., y_i's are independent; the z_j's are independent; and y_i's and z_j's are independent. Even though the process yield of different farmers within the same region may be correlated due to regional climate, we shall assume independent process yields for tractability.

When the total actual output of all n farmers is equal to O, the market selling price of the crop is equal to $(a - kO)$. By noting that each aggregation member i can generate an output of o_i units by processing q_i units, we can express the expected profit $\pi_i(s)$ for each aggregation member i $(= 1, ..., s)$ as:

$$\pi_i(s) = E\{(a - kO)o_i - cq_i\}.$$

Similarly, we can express the expected profit $\pi_j(s)$ for each individual farmer j $(= s+1, ..., n)$ who does not belong to the aggregation as:

$$\pi_j(s) = E\{(a - kO)o_j - cq_j\}.$$

By using $E(y_i) = \mu_s$ and $Var(y_i) = \sigma_s^2$ for $i = 1, ..., s$, and $E(z_j) = \mu$ and $Var(z_j) = \sigma^2$ for $j = s + 1, ..., n$, we can show that the first-order condition for each farmer can be expressed as follows:

$$\frac{\partial \pi_i}{\partial q_i} = a\mu_s - c - 2kq_i\sigma_s^2 - kq_i\mu_s^2 - k\mu_s^2 \textstyle\sum_{l=1...,s} q_l - k\mu_s\mu \sum_{l=s+1,...,n} q_l = 0 \; for \; all \; i; \quad (2.5)$$

$$\frac{\partial \pi_j}{\partial q_j} = a\mu - c - 2kq_j\sigma^2 - kq_j\mu^2 - k\mu^2 \textstyle\sum_{l=s+1,...,n} q_l - k\mu_s\mu \sum_{l=1...,s} q_l = 0 \; for \; all \; j. \quad (2.6)$$

The above equations enable us to establish the following relationship between (q_i^*) and (q_j^*) in equilibrium:

$$\frac{(2\sigma_s^2 + \mu_s^2)}{\mu_s} q_i^* - \frac{(2\sigma^2 + \mu^2)}{\mu} q_j^* = \frac{c}{k}\left(\frac{1}{\mu} - \frac{1}{\mu_s}\right) \geq 0. \quad (2.7)$$

Also, by solving (2.5) and (2.6), we get:

$$q_i^* = \frac{(a\mu_s - c)(2\sigma^2 + (n-s+1)\mu^2) - (a\mu - c)\mu_s\mu(n-s)}{k\{(2\sigma_s^2 + (s+1)\mu_s^2)(2\sigma^2 + (n-s+1)\mu^2) - s(n-s)(\mu_s\mu)^2\}} \; for \;\; i = 1, ..., \; s \quad (2.8)$$

$$q_j^* = \left(\frac{2\sigma_s^2 + \mu_s^2}{2\sigma^2 + \mu^2}\right)\frac{\mu}{\mu_s} q_i^* - \frac{c(1 - \frac{\mu}{\mu_s})}{k(2\sigma^2 + \mu^2)} \quad for \;\; j = s + 1, ..., n. \quad (2.9)$$

By examining (2.7), (2.8) and (2.9), we obtain the following result:

Lemma 1 *Suppose there is an aggregation with s members that focuses on process yield improvement. Then the following is true in equilibrium:*

(a) The process quantity of each aggregation member, q_i^*, is decreasing in σ_s^2, whereas the process quantity of each individual farmer, q_j^*, is increasing in σ_s^2.

(b) When $\frac{(2\sigma_s^2+\mu_s^2)}{\mu_s} < \frac{(2\sigma^2+\mu^2)}{\mu}$, each aggregation member i will process more than an individual farmer j in equilibrium (i.e., $q_i^* > q_j^*$).

(c) When the aggregation can help its members to reduce the variance of the process yield only so that $\mu_s = \mu$ but $\sigma_s^2 < \sigma^2$, each aggregation member i will process more than an individual farmer j in equilibrium (i.e., $q_i^* > q_j^*$).

Lemma 1 reveals that, when an aggregation enables its aggregation members to gain a competitive edge through a lower variance in process yield, aggregation members can afford to process more as σ_s^2 decreases in equilibrium. However, to avoid further reduction in the market price, other individual farmers will process less in equilibrium.

Due to the complexity of (2.8) and (2.9), let us consider a specific functional form of μ_s and σ_s^2 so that we can generate additional insights. To illustrate, consider the case when the aggregation can only reduce the variance of the process yield. Specifically, we set $\mu_s = \mu = 1$ and $2\sigma_s^2 = \frac{\beta}{s} - 1$ so that σ_s^2 is convex-decreasing in s, where β ($\geq s$) captures the uncertainty level of production yield.[6] By substituting $\mu_s = \mu = 1$ and $2\sigma_s^2 = \frac{\beta}{s} - 1$, and $2\sigma^2 = \beta - 1$ into (2.8) and (2.9), we get:

$$q_i^* = \frac{s(a-c)}{k(\beta+s^2-s+n)} \tag{2.10}$$

$$q_j^* = \frac{(a-c)}{k(\beta+s^2-s+n)} \tag{2.11}$$

Also, it is easy to check that the expected total output $O^* = s\mu_s q_i^* + (n-s)\mu q_j^* = \frac{(a-c)(s^2-s+n)}{k(\beta+s^2-s+n)}$. From (2.10) and (2.11), we note that $q_i^* \geq q_j^*$ for $s \geq 1$, which verifies Lemma 1 (c).

[6] We assume that β is sufficiently small so that we avoid the case of negative yield, i.e., $Prob(y_i < 0) \approx 0$ for all i and $Prob(z_j < 0) \approx 0$ for all j.

Next, let us examine the impact of the aggregation on the market as a whole. By noting that the total expected output in equilibrium is $O^* = s\mu_s q_i^* + (n-s)\mu q_j^* = \frac{(a-c)(s^2-s+n)}{k(\beta+s^2-s+n)}$ and the expected market selling price is $p^* = a - kO^* = a - \frac{(a-c)(s^2-s+n)}{(\beta+s^2-s+n)}$, it is easy to check the following:

Corollary 1 *Suppose there is an aggregation with s members that focuses on reducing the variance of the process yield so that $\mu_s = 1$ and $2\sigma_s^2 = \frac{\beta}{s} - 1$. Then the total expected output in equilibrium O^* is increasing in s, while the expected market price in equilibrium p^* is decreasing in s.*

Corollary 1 reveals that, when an aggregation focuses on reducing the variance of yield, the market can benefit from the existence of an aggregation in terms of higher production and lower price. Furthermore, Corollary 1 shows that the total expected output O^* increases as the aggregation size s increases. This result is intuitive because, as the aggregation size s increases, the variance of the process yields for the aggregation members decreases. Consequently, the total expected output of the aggregation members $sE(y_i q_i)$ increases in the aggregation size s. In response to this increase in process quantity of the aggregation members, other individual farmers will process less to avoid further reduction in the market price. Essentially, one can check from (2.10) and (2.11) that the increase in the total expected output of the aggregation members $sE(y_i q_i)$ dominates the decrease in the total expected output of other individual farmers $(n-s)E(z_j q_j)$, resulting in an increase of the total expected output when the aggregation size s increases.

Finally, let us examine the size of the aggregation in equilibrium. By substituting the equilibrium outputs q_i^*, q_j^*, O^* into the profit function $\pi_i(s)$ and $\pi_j(s)$, we get:

Corollary 2 *When $\mu_s = 1$ and $2\sigma_s^2 = \frac{\beta}{s} - 1$, the following results hold:*

(a) When the aggregation membership is open, each farmer is always better off joining the aggregation in the equilibrium (i.e., $\pi_i(s) > \pi_j(s)$ for $s \geq 1$). Hence, the equilibrium aggregation size is equal to n.

(b) When the aggregation membership is exclusive, a farmer can be admitted to the aggregation only when the aggregation size is less than $\frac{1}{6}\{1+\sqrt{1+12(n+\beta)}\}$. Hence, the equilibrium aggregation size $s^* = \frac{1}{6}\{1+\sqrt{1+12(n+\beta)}\}$.

Corollary 2 reveals that all n farmers will join the aggregation under open membership. Under exclusive membership, the equilibrium aggregation size s^* is increasing in the degree of yield uncertainty β.

2.5 Focusing on improving brand awareness

An aggregation can help its members to increase demand by creating brand awareness (e.g., launching promotion campaign using "cause marketing"). For example, Lijjat Papad (www.lijjat.com) of Mumbai founded in 1959 is a coop that focuses on helping women to overcome hardship. This coop manufactures and sells various products including Papad, Appalam, and Masala. Because many people are aware of the mission of Lijjat Papad, its brand is well-recognized throughout India. Besides Lijjat Papad, Amul is another successful coop founded in 1946 in India (www.amul.com). Not only are Amul is the largest dairy coop in India, it has promoted its brands in USA, Australia, China, Singapore and Hong Kong. Finally, in the developed economies, Fonterra of New Zealand (www.fonterra.com) is the largest dairy coop with $10,500$ members producing 30% of the world's dairy products.

To model consumer brand awareness of the aggregation, we consider the case when the consumer base of the products produced by the aggregation with s members is equal to $a(s)$, where $a(s)$ is increasing for $s \geq 1$ (For notational convenience, we define $a(1) = a$). Each farmer i who belongs to the aggregation with s members produces q_i, and will obtain a unit price $p_i = a(s) - kQ$ for $i = 1, 2, ..., s$, where Q is the total amount produced by all n farmers. Hence, the profit of each aggregation member i can be written as:

$$\pi_i = \{a(s) - kQ - c\}q_i \ for\ i = 1, 2, ..., s. \tag{2.12}$$

Similarly, any individual farmer j produces q_j will obtain a unit price $p_j = a - kQ$ for

$j = s+1, s+2, ..., n$, so that his profit is:

$$\pi_j = \{a - kQ - c\}q_j \; for \; j = s+1, s+2, ..., n. \tag{2.13}$$

By considering the first-order conditions associated with (2.12) and (2.13), we establish the following result:

Proposition 2 *Suppose there is an aggregation with s members that improves brand awareness. Then the production quantity of each aggregation member i in equilibrium satisfies:*

$$q_i^* = \frac{(n-s+1)(a(s)-a)+a-c}{k(n+1)} \; for \; i = 1, 2, ..., s, \tag{2.14}$$

where q_i^* is increasing in s if and only if $\frac{\partial a(s)}{\partial s} > \frac{a(s)-a}{n-s+1}$. Also, the production quantity of each individual farmer j in equilibrium satisfies:

$$q_j^* = \frac{s(a-a(s))+a-c}{k(n+1)} \; for \; j = s+1, s+2, ..., n, \tag{2.15}$$

where q_j^* is decreasing in s. Moreover, $q_i^* \geq q_j^*$ and $\pi_i(s) \geq \pi_j(s)$ in equilibrium for any given $s \geq 1$

Because each aggregation member enjoys a higher selling price by increasing brand awareness, Proposition 2 suggests that each aggregation member i will always produce more and earn larger profit than an individual farmer j in equilibrium.

Now, let us examine the impact of the aggregation on the market as a whole. In equilibrium, the total output is $Q^* = sq_i^* + (n-s)q_j^*$, and the market price for an aggregation member i is $p_i^* = a(s) - kQ^*$, and the market price for an individual farmer j is $p_j^* = a - kQ^*$. By substituting q_i^* and q_j^* into these results, we show that the total output in equilibrium is $Q^* = \frac{s(a(s)-a)+n(a-c)}{k(n+1)}$, which is increasing in s. Also, the market price for an aggregation member $p_i^* = \frac{(n-s+1)(a(s)-a)+a(n+1)-n(a-c)}{n+1}$ is increasing in s if and only if $a(s)$ is increasing fast enough; i.e., $\frac{\partial a(s)}{\partial s} > \frac{a(s)-a}{n-s+1}$; however, the market price for an individual farmer $p_j^* = \frac{a+nc-s(a(s)-a)}{n+1}$ is decreasing in s. Moreover, $p_i^* \geq p_j^*$ for $s \geq 1$. This result reveals that the market as a whole can benefit from the existence of an aggregation in terms of higher quantity. However, in terms of market price, the result is mixed. Specifically, the impact of the aggregation on the member's selling price p_i^* is non-monotonic, while individual farmer's selling price p_j^* is monotonically decreasing in s. Hence, when the aggregation increases its brand awareness, the market price of the product produced by the aggregation members can be higher. However, due to market competition, the market price of the product produced by other individual farmers is actually lower.

Finally, let us examine the size of the aggregation in equilibrium. Proposition 2 reveals that, when the admission option is available to all farmers (i.e., under open membership), it is beneficial for all farmers to join the aggregation so that the equilibrium aggregation size is equal to n. However, under the exclusive membership, we can determine the equilibrium size of the aggregation s^* by maximizing an aggregation member's profit $\pi_i(s)$. From (2.14) and (2.15), we can conclude the following:

Corollary 3 *Suppose there is an aggregation with s members that improves brand awareness. Then under exclusive membership, the equilibrium aggregation size s^* satisfies* $\frac{\partial a(s)}{\partial s}\Big|_{s=s^*} = \frac{a(s^*)-a}{n-s^*+1}$

To illustrate, let us examine the case when $a(s)$ is concave-increasing in s to reflect the case when the marginal benefit in brand awareness from adding a new member to the aggregation decreases as the aggregation size increases. The same analysis can be used to analyze the case when $a(s)$ is convex-increasing to capture the potential "network effect" (i.e., the consumer base grows exponentially as the size of the aggregation grows). By applying Corollary 3 for the case when $a(s) = a\sqrt{s}$, the equilibrium aggregation size satisfies: $s^* = \frac{1}{9}(3n + 5 - 2\sqrt{3n+4})$. Clearly, s^* is increasing in n. This is consistent with the results presented in Section 2.3 and 2.4 for the aggregation that reduces marginal cost and yield uncertainty, respectively.

2.6 Focusing on improving selling prices using direct channel

We now examine a situation in which an aggregation that helps members to increase their selling price through a direct channel that is often run by a social enterprise. Consider the coconut farmers in the Philippines who usually sell their coconut products through many intermediaries and get only a small fraction of the final market selling price for their crop. Because these farmers do not have the requisite knowledge, experience or resources to sell direct, the best alternative is to sell through a social enterprise by eliminating all other non-value added middlemen so that the farmers can get a higher price. For example, a for-profit social enterprise Coconut World is a distributor/retailer who buys coconut sugar (and other coconut products) directly from farmers in the Philippines and sells directly to retailers in the US. By shortening the supply chain, the farmers get a higher selling price ([CHT10]). Besides social enterprises, retail giants such as Walmart, Carrefour and Metro have various initiatives that are intended to cut the middlemen so that farmers can sell their produce at a higher price. For example, Walmart launched its Direct Farm program in China under which Walmart purchases the crops directly from farmers.[7] By cutting the middlemen, farmers can obtain a higher price and Walmart can reduce their cost and improve the freshness of the produce.

While it is clear that the farmers should get a higher selling price when all farmers are selling in a supply chain through one distributor (e.g., a social enterprise), it is not clear if this is the case when there are two competing supply chains operating in the same market: one supply chain has s aggregation members selling through one distributor, and a competing supply chain has $(n - s)$ individual farmers selling through a supply chain with m layers of middlemen. To ease our exposition, we shall consider the case when $m = 2$.[8]

Consider two competing supply chains. In the first supply chain, there is an aggregation of s (≥ 1) farmers who first sets its unit price r, and then sells its crop directly to a single

[7] Since the launch of Walmart's Direct Farm program in 2007, over 700,000 farmers have participated in 2010 (See www.wal-martchina.com for details).

[8] One can consider the case where $m > 2$ but this complicates the analysis significantly without providing additional insight.

social enterprise. Given the unit price r, the social enterprise determines the quantity q_i to be purchased from each aggregation member i so that sq_i will be sold in the market by the first supply chain. In the second supply chain, there are $(n-s)$ individual farmers who set their unit price w_1 and sell their crop to Intermediary 1. Given w_1, Intermediary 1 sets it unit price w_2 and sells the crop to Intermediary 2. Given the unit price w_2, Intermediary 2 determines $(n-s)q_j$, the total amount to be purchased from Intermediary 1, which is the amount to be sold in the market by the second supply chain. As the social enterprise and intermediary 2 engage in Cournot competition, the market price is equal to $p = a - bQ$, where $Q = sq_i + (n-s)q_j$ represents the total quantity available for sales in the market. Figure 2.2 depicts the two competing supply chain as described.

We now analyze these two supply chains using backward induction. First, we analyze the equilibrium outcomes of the social enterprise and intermediary 2 for any given selling prices set by the farmers and intermediary 1; i.e., (r, w_1, w_2). Second, we determine the equilibrium outcomes of intermediary 1 for any given selling prices set by the farmers, (r, w_1). Third, we analyze the equilibrium outcomes of the farmers (i.e., aggregation members and other individual farmers).

To begin, for any given selling prices (r, w_1, w_2), it is easy to check from the model description above that the social enterprise's profit is equal to $\Pi_0 = (a - bQ - r)sq_i$ and intermediary 2's profit is equal to $\Pi_2 = (a - bQ - w_2)(n-s)q_j$, where $Q = sq_i + (n-s)q_j$. By considering the derivative of Π_0 with respect to q_i and the derivative of Π_2 with respect to q_j, the equilibrium outcomes satisfy:

$$q_i^* = \frac{a-2r+w_2}{3bs},\ q_j^* = \frac{a-2w_2+r}{3b(n-s)},\ and\ Q^* = \frac{a-w_2-r}{3b}. \tag{2.16}$$

Next, for any given selling prices set by the farmers (r, w_1), intermediary 1's profit is equal to $\Pi_1 = (w_2 - w_1)[(n-s)q_j^*]$. By using (2.16) and by considering the derivative of Π_1 with respect to w_2, intermediary 1 will charge w_2^* in equilibrium, where:

$$w_2^* = \frac{a + r + 2w_1}{4}. \tag{2.17}$$

Finally, we analyze the equilibrium outcomes of the aggregation members and individual farmers. For each of s aggregation members, her profit is equal to $\pi_i = (r-c)q_i^*$ for $i = 1, .., s$. Similarly, for each of those $(n-s)$ individual farmers, his profit is equal to $\pi_j = (w_1 - c)q_j^*$ for $j = s+1, .., n$. By using (2.16) and (2.17), and by considering the derivative of π_i with respect to r and the derivative of π_j with respect to w_1, the prices in equilibrium satisfy:

$$r^* = \tfrac{11}{27}a + \tfrac{16}{27}c, \textit{ and } w_1^* = \tfrac{19}{54}a + \tfrac{35}{54}c. \tag{2.18}$$

By noting that $r^* - w_1^* = \frac{3}{54}(a-c) > 0$, we can conclude that, despite competition between two supply chains, aggregation members can always obtain a higher selling price by shortening their supply chain.

To examine the impact of the aggregation on the market as a whole, one can use (2.16), (2.17) and (2.18) to show that the amount to be sold by the social enterprise (i.e., the direct channel) is equal to $sq_i^* = \frac{1}{3b}(\frac{77}{108}a - \frac{77}{108}c)$ and the amount to be sold by intermediary 2 is equal to $(n-s)q_j^* = \frac{1}{3b}(\frac{19}{54}a - \frac{19}{54}c)$, where both quantities are independent of the size of the aggregation s in equilibrium. Consequently, the total output $Q^* = sq_i^* + (n-s)q_j^* = \frac{s}{3bs}(\frac{77}{108}a - \frac{77}{108}c) + \frac{(n-s)}{3b(n-s)}(\frac{19}{54}a - \frac{19}{54}c) = \frac{1}{3b}(\frac{115}{108})(a-c)$ and the market price $p^* = a - kQ^*$ are also independent of s. Therefore, when the aggregation sells through a direct channel, it has no impact on the market because the total output and the market price in equilibrium are independent of s.

Next, let us compare the profit in equilibrium of aggregation and other individual farmers. By using (2.18) and retrieving the equilibrium outcomes by backward substitutions through (2.16) and (2.17), we can conclude the following:

Proposition 3 *Suppose there is an aggregation with s members that uses a direct channel to sell. Then the followings are true in equilibrium:*

(a) An aggregation member will produce more than an individual farmer if and only if $s < \frac{77}{115}n \approx 0.67n$ (i.e., $q_i^* > q_j^*$ if and only if $s < 0.67n$); and

(b) An aggregation member will earn more than an individual farmer if and only if $s < 0.7n$ (i.e., $\pi_i(s) > \pi_j(s)$ if and only if $s < 0.7n$).

By recalling from above that $r^* > w_1^*$, one would expect the aggregation members will produce less to support a higher selling price. However, as the aggregation size s increases, it is easy to check from the above that each aggregation member would need to reduce his quantity sold q_i^* to avoid price erosion (i.e., $q_i^* = \frac{1}{3bs}(\frac{77}{108}a - \frac{77}{108}c)$ is decreasing in the aggregation size s) and each individual farmer would respond by increasing his amount sold $q_j^* = \frac{1}{3b(n-s)}(\frac{19}{54}a - \frac{19}{54}c)$, which is increasing in the aggregation size s. This explains Proposition 3 (a): an aggregation member would sell more than an individual farmer when the aggregation size is sufficiently small.

Next, let us examine the size of the aggregation in equilibrium under open membership. To begin, recall from (2.18) that the aggregation members can obtain a higher selling price r^* than other individual farmers w_1^*; i.e., $r^* > w_1^*$, where r^* and w_1^* are independent of s. Also, recall from the above $q_i^* = \frac{1}{3bs}(\frac{77}{108}a - \frac{77}{108}c)$ is decreasing s, while $q_j^* = \frac{1}{3b(n-s)}(\frac{19}{54}a - \frac{19}{54}c)$ is increasing in s. These observations implies that the profit of each aggregation member $\pi_i = (r^* - c)q_i^*$ is decreasing in s, while the profit of each individual farmer $\pi_j = (w_1^* - c)q_j^*$ is increasing in s. By comparing π_i and π_j, Proposition 3(b) reveals that joining the aggregation is beneficial as long as $s < 0.7n$; i.e., $\pi_i(s) > \pi_j(s)$ if and only if $s < 0.7n$. Hence, the equilibrium aggregation size is equal to $0.7n$. This result is in contrast to the results obtained from the other models presented earlier in Section 2.3, 2.4, and 2.5 that all farmers will join the aggregation under open membership. This result may help explaining why some farmers do not join the aggregation in practice as observed by Coconut World ([CHT10]).

Under exclusive membership, the equilibrium aggregation size $s^* = 1$ because the aggregation member's profit $\pi_i(s) = \frac{1}{3bs}(\frac{77\times 11}{27\times 108})(a-c)^2$ is decreasing in s. This suggests that only one farmer would sell through the social enterprise by blocking the entry of other farmers. Hence, we suggest that the aggregation focusing on improving selling prices using direct channel should be operated under open membership so as to prevent that only one farmer can enjoy the entire benefits of the aggregation alone under exclusive membership.

2.7 Focusing on reducing selling price risks

We now examine a situation in which each farmer can either: (a) join an aggregation with s members and sell directly to a risk-neutral firm that offers a guaranteed unit price $p_i = \alpha - \beta q_i$ for $i = 1, 2, ..., s$ where q_i is the process quantity of an aggregation member i;[9]or (b) refuse to join the aggregation and sell in the open market with uncertain unit price $p_j = a - bQ + \epsilon$ for $j = s + 1, s + 2, ..., n$ that depends on uncertain market condition ϵ, where $\epsilon\ thicksim\ Normal(0, \sigma^2)$ and $Q = \sum_{l=s+1}^{n} q_l$.[10] This setting resembles the operations of the e-Choupal initiative launched by ITC (www.itcportal.com) in 2000 that was intended to help soya bean farmers in Madhya Pradesh to obtain a fair and stable selling price of their crops.[11] Specifically, ITC works with villagers to select a trained representative who helps them to access ITC's web portal (via a kiosk) in order to learn of the commodity prices traded at various mandis (open markets) in different locations in the previous day. At the same time, ITC announces on its portal its offer to purchase the produce from the farmer at a fixed price. This way, each farmer has two options: (a) sell to ITC according to the pre-announced price ([ST12]), or (b) sell at the mandi at an uncertain market price in the next day.

Consider n identical *risk-averse* farmers that is commonly assumed in development economics literature ([CC08]). For any wealth z, each farmer's utility function $U(z) = 1 - e^{-\rho z}$, where $\rho > 0$. For each aggregation member i who produces and sells q_i directly to the risk-neutral firm at a pre-announced unit price $p_i = \alpha - \beta q_i$, his profit is equal to $\pi_i = (\alpha - \beta q_i - c) q_i$ and his utility is equal to $U(\pi_i) = 1 - e^{-\rho[(\alpha - \beta q_i - c) q_i]}$, where $i = 1, ..., s$. In this case, maximizing the utility of an aggregation member is the same as maximizing $CE_i = (\alpha - \beta q_i - c) q_i$,

[9]When there is an aggregation at the upstream tier of the supply chain selling to risk-neutral firms at the second tier, then we can obtain a linear demand function as shown in [CK01].

[10]Technically, a firm can purchase directly with a single farmer without any aggregation requirement. However, in most instances, this firm is a large multi-national company who buys crops as basic ingredients for their products. Due to the scale of its operations, the firm usually deals with the aggregation so that the scale of the transaction is large enough to justify direct purchases. For example, under the "Creating Shared Value" initiative, Nestle is committed to help poor farmers economically. They now buy coffee beans directly from aggregation members in Ivory Coast under an exclusive arrangement at a guaranteed price. The reader is referred to Lee et al. (2011) for details.

[11]By 2010, ITC's e-Choupal initiative has empowered over 4 million farmers in 4000 villages located in 10 different states in India.

where CE_i is known as the "certainty equivalence". By considering the first derivative, the optimal production and sales quantity of each aggregation member i is $q_i^* = \frac{\alpha-c}{2\beta}$ and the optimal $CE_i^* = \frac{(\alpha-c)^2}{4\beta}$, where $i = 1, 2, ..., s$. Next, for each individual farmer $j (= s+1, s+2, ..., n)$ who does not belong to the aggregation, farmer j will sell its crop in the open market with uncertain market price. When the total amount to be sold in the open market is equal to $Q = \sum_{l=s+1}^{n} q_l$, the market price is equal to $(a - bQ + \epsilon)$. Farmer j's profit is equal to $\pi_j = (a - bQ + \epsilon - c)q_j$ and his expected utility is equal to $E(U_j) = 1 - E(e^{-\rho[(a-bQ+\epsilon-c)q_j]}) = 1 - e^{-\rho[(a-bQ-c)q_j - \frac{\rho}{2}\sigma^2(q_j)^2]}$. The term $CE_j = (a - bQ - c)q_j - \frac{\rho}{2}\sigma^2(q_j)^2$ is known as the certainty equivalence, and maximizing the expected utility $E(U_j)$ is the same as maximizing CE_j. Hence, we focus on CE_j. By considering the best response of each individual farmer, we can determine the equilibrium quantity q_j^* of each individual farmer j as $q_j^* = \frac{a-c}{b(n-s+1)+\rho\sigma^2}$. By substituting q_j^* into CE_j, the equilibrium certainty equivalence CE_j^* can be expressed as: $CE_j^* = (\frac{\rho}{2}\sigma^2 + b)(q_j^*)^2 = (\frac{\rho}{2}\sigma^2 + b)[\frac{a-c}{b(n-s+1)+\rho\sigma^2}]^2$. We summarize the production quantities and the certainty equivalences in equilibrium for the aggregation members and other individual farmers in Table 2.1.

Based on the equilibrium outcomes summarized in Table 2.1, we can compare the selling prices of the aggregation members and other individual farmers as well as the certainty equivalences. First, the unit selling price of each aggregation member i is equal to $p_i^* = \alpha - \beta q_i^* = \frac{\alpha+c}{2}$. Second, the expected unit selling price of each individual farmer j is equal to $E(p_j^*) = a - bQ^* = a - b(n-s)q_j^* = a - b(n-s)[\frac{a-c}{b(n-s+1)+\rho\sigma^2}]$, which is increasing in σ. Hence, each aggregation farmer obtains a higher selling price only when the open market price uncertainty σ is sufficiently low. More formally, we have:

Proposition 4 *Suppose there is an aggregation with s members that focuses on reducing selling price risks. Then aggregation member i obtains a higher selling price than an individual farmer j if and only if $\sigma^2 < \frac{b}{\rho}\{\frac{((\alpha-c)(n-s)}{2a-\alpha-c} - 1\}$. In other words, $p_i^* > E(p_j^*)$ if and only if $\sigma^2 < \frac{b}{\rho}\{\frac{((\alpha-c)(n-s)}{2a-\alpha-c} - 1\}$.*

Proposition 4 can be explained as follows. When σ is sufficiently high, each risk-averse individual farmer j would produce less to reduce the risk of selling at a lower price; i.e., q_j^* is decreasing in σ. Consequently, individual farmers' market price is greater than the aggregation members' when σ is sufficiently high. This observation implies that the aggregation member will obtain a higher price than the individual farmer only when σ is sufficiently low. Consider a special case in which the risk-neutral firm offers a guaranteed unit price $p_i = \alpha - \beta q_i$ that has $\alpha = a$, and $\beta = b$. (This situation is essentially the case when the risk-neutral firm absorbs the price uncertainty.) In this case, the threshold $\frac{b}{\rho}\{\frac{((\alpha-c)(n-s)}{2a-\alpha-c} - 1\}$ is reduced to $\frac{b}{\rho}(n-s-1)$ and $p_i^* > E(p_j^*)$ if and only if $\sigma^2 < \frac{b}{\rho}(n-s-1)$. Also, observe that the threshold $\frac{b}{\rho}\{\frac{((\alpha-c)(n-s)}{2a-\alpha-c} - 1\}$ is decreasing in s. This implies $p_i^* > E(p_j^*)$ occurs when the size of the aggregation is sufficiently large.

Next, we examine and compare the certainty equivalences of aggregation members and other individual farmers in equilibrium. By considering the special case when $\alpha = a$, and $\beta = b$ and by utilizing the results in Table 2.1, we can show that $CE_i^* > CE_j^*$ if and only if $[b(n-s)+\rho\sigma^2]^2 > b^2[3-2(n-s)]$. By noting this condition holds when $s \leq n-1$ for any value of $0 < \sigma^2 \leq \frac{b}{\rho}\sqrt{3}$, and it holds when $\sigma^2 > \frac{b}{\rho}\sqrt{3}$ for any value of s, we can draw the following conclusion. First, when price uncertainty is sufficiently high (say, $\sigma^2 > \frac{b}{\rho}\sqrt{3}$), $CE_i^* > CE_j^*$ for any value of s. Hence, under open membership, the equilibrium aggregation size is equal to n. Second, when price uncertainty is low (say, $\sigma^2 \leq \frac{b}{\rho}\sqrt{3}$), the equilibrium aggregation size is equal to $(n-1)$ under open membership. Finally, under exclusive membership, it is easy to check that existing aggregation members will not oppose to the admission of new members because the certainty equivalence of each aggregation member $CE_i^* = \frac{(\alpha-c)^2}{4\beta}$ is independent of the size of the aggregation s in equilibrium. Hence, new member will join the aggregation only when $CE_i^* > CE_j^*$. Consequently, the equilibrium aggregation size under exclusive membership is the same as in the case of open membership. Throughout Section 2.3 to Section 2.7, we have characterized the equilibrium outcomes including strategic quantity decisions, profits, market price, total production in the market, and the equilibrium aggregation sizes under both open and exclusive membership. We further highlight the insights of the results in Section 2.8.

2.8 Concluding Remarks

In this chapter, we have investigated the five operational effects created by an aggregation, and characterized the strategic quantity decisions of aggregation members and other individual farmers for each of the five effects. Further, we have examined the impact of these effects of an aggregation on the market as a whole in terms of market price and total production quantity. Even though we analyze separate models so as to isolate the impact of each effect, our models can serve as building blocks for analyzing the case when multiple effects are present simultaneously in a single aggregation. However, the analysis of a single model that deals with multiple effects simultaneously is complex and we shall defer such analysis to future research.

We have shown that most of the effects of an aggregation are beneficial to the market as a whole in terms of lower market price and higher total production, but other effects are hurtful to the market in terms of higher market price (e.g., the aggregation focusing on improving brand awareness). When certain aggregation effects hurt the market, higher market price can help farmers break their poverty cycle. Besides aggregations, governments in developing countries have instituted various policies (quotas or tariffs for importing or exporting various commodities) to prevent price erosion ([GWP04]). However, as shown in the above sections, agricultural cooperatives can be an alternative way to improve farmers' earnings without government interventions.

We have also provided the conditions under which the farmers should join an aggregation in each of the five effects. Based on our analysis, we have observed a general pattern: it is beneficial for a farmer to join the aggregation only when the size of the aggregation is below a certain threshold. This result can be intuitively explained as follows: One would expect that each of the five operational effects can boost each aggregation member's profit. However, as the size of the aggregation increases, this benefit diminishes so that the incentive to admit a new member decreases. This explains why there is a critical size after which the admission of new members has an adverse effect on the existing members' profit.

Further, it is worth noting that the equilibrium size of the aggregation would be different depending on the type of cooperative formation: (1) open membership: farmers are free to join or leave the aggregation; or (2) exclusive membership: the admission process requires the approval of existing aggregation members. Further, our analysis of the equilibrium aggregation size explains why not all farmers join an aggregation (or a coop) in practice even when each effect is beneficial to aggregation members. For example, the aggregation focusing on improving selling prices using direct channel (Section 2.6) can explain this counter-intuitive phenomenon observed in practice (only a fraction of farmers join an aggregation even under open membership). This can be one of the reasons why we do not observe the grand coalition (i.e., all n farmers join the aggregation) in practice.

Our models have several limitations that deserve further investigation. First, because we examine different effects through separate models, an obvious extension would be to consider ways to analyze the value created by combining different aggregation effects simultaneously. To analyze the value of combining different aggregation effects, one needs to construct a unified model which incorporates multiple effects so as to examine the relationships among the effects analytically. Second, note that our analysis is based on the assumption that all farmers are identical. Hence, it is of interest to extend our model to the case when farmers are non-identical even though the equilibrium may not be unique. Third, by noting that our analysis is limited to the case when there is only one aggregation (or coop), it is of interest to consider the case when the number of aggregations in the market is determined *endogenously*. Such analysis would enable us to characterize the aggregation structure in equilibrium. Fourth, due to the decentralized decision making of the members of aggregation, one can expect there would be free-riding problem on production quantity and quality. Hence, extending our models to incorporate the issue of free-riding and self-interest could result in newer insights. Finally, while our models focus on the operational effects associated with aggregation, there are other issues such as corruption and bureaucracy that tend to happen in emerging economies deserve further examination.

Appendix 2.1

In this appendix, we describe how our model for the cost reduction case in Section 2.3 can be extended to the case when an admission fee $f(>0)$ is required for a farmer to join an aggregation. The modified profit function of each aggregation member $i(=1,2,...,s)$ becomes $\tilde{\pi}_i(s) = \pi_i(s) - f$, where $\pi_i(s)$ is stated in Section 2.3. The modified profit function of each individual farmer $j(=s+1, s+2, ..., n)$ becomes $\tilde{\pi}_j(s) = \pi_j(s)$ where $\pi_j(s)$ is stated in Section 2.3. By considering the first order conditions, we can show that the modified process quantity in equilibrium is given as $\tilde{q}_i^* = \frac{a+nc-(s-1)c}{k(n+1)} - \frac{c}{ks}$ for all i and $\tilde{q}_j^* = \frac{a+nc-(s-1)c}{k(n+1)} - \frac{c}{k}$ for all j. It is easy to see that $\tilde{q}_i^* = q_i^*$ and $\tilde{q}_j^* = q_j^*$ for all i, j where q_i^* and q_j^* are stated in Section 2.3. Furthermore, the modified total output $\tilde{Q}^* = s\tilde{q}_i^* + (n-s)\tilde{q}_j^*$ and the modified market price $\tilde{p}^* = a - k\tilde{Q}^*$ also remain the same (i.e., $\tilde{Q}^* = Q^*$ and $\tilde{p}^* = p^*$ where Q^* and p^* are stated in Section 2.3). Hence, we conclude that the equilibrium outcomes including process quantities, market price, and total output remain unchanged even if we introduce a positive admission fee to the model.

However, the equilibrium size of the aggregation under open membership can be different because the equilibrium profit of an aggregation member is not necessarily larger than that of an individual farmer. Specifically, a farmer will join the aggregation if $\tilde{\pi}_i(s) > \tilde{\pi}_j(s)$ under open membership. This condition is equivalent to $\pi_i(s) - \pi_j(s) > f$. Thus, if the admission fee is sufficiently high (i.e., $f > \pi_i(s) - \pi_j(s)$), no farmers will join the aggregation under open membership. This result is different from the result in Section 2.3 where all farmers join the aggregation under open membership without the admission fee. Under exclusive membership, we can determine the equilibrium size $\tilde{s}^*$ by maximizing an aggregation member's profit $\tilde{\pi}_i(s)$. Since $\tilde{\pi}_i(s) = \pi_i(s) - f$, maximizing $\tilde{\pi}_i(s)$ is the same as maximizing $\pi_i(s)$. Hence, it is easy to show that $\tilde{s}^* = \sqrt{n+1}$ which is the same as s^* stated in Section 2.3. Therefore, the equilibrium size under exclusive membership remains the same.

Appendix 2.2: Proofs

Proof of Proposition 1 From (2.1) and (2.2), it is easy to see that profit functions $\pi_i(s)$ and $\pi_j(s)$ are concave in q_i and q_j, respectively. Hence the first order condition is necessary and sufficient to obtain the equilibrium process quantities. Taking the first derivatives of (2.1) and (2.2) yields $\frac{\partial \pi_i(s)}{\partial q_i} = a - kQ - \frac{c}{s} - kq_i = 0$ for $i = 1, 2, ..., s$ and $\frac{\partial \pi_j(s)}{\partial q_j} = a - kQ - c - kq_j = 0$ for $j = s+1, s+2, ..., n$. We obtain a unique solution to this set of equations given by $q_i^* = \frac{a+nc-(s-1)c}{k(n+1)} - \frac{c}{ks}$ for $i = 1, 2, ..., s$ and $q_j^* = \frac{a+nc-(s-1)c}{k(n+1)} - \frac{c}{k}$ for $j = s+1, s+2, ..., n$. Then, $\frac{\partial q_i^*}{\partial s} = \frac{c}{k}(\frac{1}{s^2} - \frac{1}{n+1})$, which is strictly positive if and only if $s < \sqrt{n+1}$. Also, it is obvious that q_j^* is decreasing in s from (2.4). Finally, rearranging terms from (2.3) and (2.4) shows $q_i^* - q_j^* = \frac{c}{k}(1 - \frac{1}{s}) \geq 0$ for any $s \geq 1$. By substituting the first order conditions into the profit functions, we obtain $\pi_i = k(q_i^*)^2$ and $\pi_j = k(q_j^*)^2$. Consequently, we can conclude $\pi_i \geq \pi_j$ in equilibrium for any $s \geq 1$.

Proof of Lemma 1 Observe that q_i^* is decreasing in σ_s^2 from (2.8). Also, because

$$q_j^* = \frac{\mu}{\mu_s(2\sigma^2 + \mu^2)} \frac{(a\mu_s - c)(2\sigma^2 + (n-s+1)\mu^2) - (a\mu - c)\mu_s\mu(n-s)}{k\{2\sigma^2 + (n-s+1)\mu^2 + \frac{s\mu_s^2(2\sigma^2+\mu^2)}{2\sigma_s^2+\mu_s^2}\}} - \frac{c(1 - \frac{\mu}{\mu_s})}{k(2\sigma^2 + \mu^2)},$$

it is easy to see that q_j^* is increasing in σ_s^2. This proves part (a). The results stated in part (b) and (c) follow immediately from (2.7).

Proof of Corollary 1 We can rewrite the total expected output $O^* = \frac{(a-c)(s^2-s+n)}{k(\beta+s^2-s+n)}$ as $O^* = \frac{(a-c)}{k(1+\frac{\beta}{s^2-s+n})}$. Since (s^2-s+n) is increasing in $s(\geq 1)$, O^* is increasing in s. Consequently, the expected market price $p^* = a - kO^*$ is decreasing in s.

Proof of Corollary 2 Substituting q_i^*, q_j^*, and O^* into $\pi_i(s) = E\{(a - kO^*)y_i q_i^* - cq_i^*\}$ and $\pi_j(s) = E\{(a - kO^*)z_j q_j^* - cq_j^*\}$ yields $\pi_i(s) = \frac{s\beta}{k}(\frac{a-c}{\beta+s^2-s+n})^2$ and $\pi_j(s) = \frac{\beta}{k}(\frac{a-c}{\beta+s^2-s+n})^2$. It is easy to see that $\pi_i(s) \geq \pi_j(s)$ for $s \geq 1$. Hence, under open membership, each farmer is better off by joining the aggregation. Next, under exclusive membership, we can determine the equilibrium size of the aggregation s^* by maximizing the aggregation member's profit $\pi_i(s)$. By considering the condition in which β is sufficiently small to ensure that $Prob(y_i < 0) \approx 0$ for all i and $Prob(z_j < 0) \approx 0$ for all j, it is easy to show that the second derivative of

$\pi_i(s) = \frac{s\beta}{k}(\frac{a-c}{\beta+s^2-s+n})^2$ with respect to s is negative for $s \geq 1$. This implies $\pi_i(s)$ is concave in $s(\geq 1)$. Hence, by considering the first order condition, it is easy to check that the equilibrium aggregation size $s^* = \frac{1}{6}\{1 + \sqrt{1 + 12(n+\beta)}\}$.

Proof of Proposition 2 From (2.12) and (2.13), we compute the first order condition $\frac{\partial \pi_i}{\partial q_i} = a(s) - kQ - c - kq_i = 0$ and $\frac{\partial \pi_j}{\partial q_j} = a - kQ - c - kq_j = 0$. Solving the system of linear equations yields $q_i^* = \frac{(n-s+1)(a(s)-a)+a-c}{k(n+1)}$ and $q_j^* = \frac{s(a-a(s))+a-c}{k(n+1)}$. Taking the first derivative of q_i^* with respect to s shows that $\frac{\partial q_i^*}{\partial s} > 0$ if and only if $\frac{\partial a(s)}{\partial s} > \frac{a(s)-a}{n-s+1}$. This implies q_i^* is increasing in s if and only if $\frac{\partial a(s)}{\partial s} > \frac{a(s)-a}{n-s+1}$. Further, it is easy to see that q_j^* is decreasing in s, and $q_i^* \geq q_j^*$ given that $a(s) \geq a$ from (2.12) and (2.13). Finally, we have $\pi_i \geq \pi_j$ because $\pi_i = k(q_i^*)^2$ and $\pi_j = k(q_j^*)^2$.

Proof of Corollary 3 Since $\pi_i(s) = k(q_i^*)^2$, it suffices to maximize q_i^* for maximizing $\pi_i(s)$. After some algebra, we obtain $\frac{\partial^2 q_i^*}{\partial s^2} = \frac{\partial^2 a(s)}{\partial s^2}(n-s+1) - 2\frac{\partial a(s)}{\partial s}$, which is always negative because $a(s)$ is concave-increasing in s (i.e., $\frac{\partial a(s)}{\partial s} > 0$ and $\frac{\partial^2 a(s)}{\partial s^2} < 0$). Therefore, $\pi_i(s)$ is maximized at s^* that s^* satisfies the first order condition (i.e., $\frac{\partial q_i^*}{\partial s} = 0 \iff \frac{da(s)}{ds}_{s=s^*} = \frac{a(s^*)-a}{n-s^*+1}$).

Proof of Proposition 3 By retrieving all equilibrium outcomes by backward substitutions through (2.17) and (2.16), we have $w_2^* = \frac{57}{108}a + \frac{51}{108}c$ and $q_i^* = \frac{1}{3bs}(\frac{77}{108}a - \frac{77}{108}c)$ and $q_j^* = \frac{1}{3b(n-s)}(\frac{19}{54}a - \frac{19}{54}c)$. Rearranging terms shows that $q_i^* > q_j^*$ if and only if $s < \frac{77}{115}n$. Further, $\pi_i = (r^* - c)q_i^* = \frac{1}{3bs}(\frac{77\times 11}{27\times 108})(a-c)^2$ for $i = 1, .., s$ and $\pi_j = (w_1^* - c)q_j^* = \frac{1}{3b(n-s)}(\frac{19}{54})^2(a-c)^2$ for $j = s+1, .., n$. By rearranging terms, it is easy to see that $\pi_i > \pi_j$ if and only if $s < 0.7n$.

Proof of Proposition 4 We obtained the desired results by comparing $p_i^* = \frac{\alpha+c}{2}$ and $E(p_j^*) = a - b(n-s)[\frac{a-c}{b(n-s+1)+\rho\sigma^2}]$.

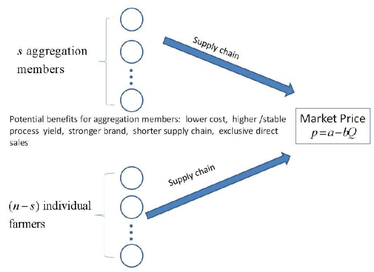

Figure 2.1: Unified Framework

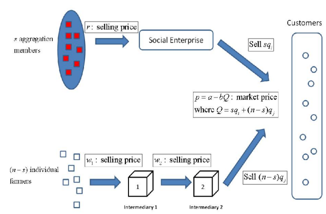

Figure 2.2: Supply Chain Configuration

Table 2.1: Equilibrium Outcomes

	Aggregation Member i	Individual Farmer j
Production Quantity	$q_i^* = \frac{\alpha - c}{2\beta}$	$q_j^* = \frac{a-c}{b(n-s+1)+\rho\sigma^2}$
Certainty Equivalent	$CE_i^* = \frac{(\alpha-c)^2}{4\beta}$	$CE_j^* = (\frac{\rho}{2}\sigma^2 + b)[\frac{a-c}{b(n-s+1)+\rho\sigma^2}]^2$

CHAPTER 3

Fractional Price Matching Policies Arising from the Ocean Freight Service Industry

3.1 Introduction

Ocean freight continues to play the most significant role in supporting material flows along global supply chains. In fact, nearly 85% of international trade involves ocean transportation ([YY12]). According to Merge Global Analysis and Estimate (Merge Global 2008), the total revenue of ocean freight industry is over $200 billion in 2007. Even though ocean transportation is well studied research area in maritime economics, [FL12] commented that there are many supply chain management issues arising from ocean freight are not well-understood. Specifically, the issue of pricing and contracting arising from ocean freight is an emergent topic that practitioners have begun to examine. While the issue of dynamic pricing has been well studied in the revenue management literature that is motivated by the commercial airline industry in a B2C environment, the pricing and contracting issues arising from the ocean transportation industry in a B2B environment are not well understood. Unlike the commercial airline industry that has a well-established system to deal with issues such as dynamic pricing, over-booking, etc., the ocean transportation industry lacks a systematic approach for examining the implications of pricing or contracting issues.

In general, there are two major players in the ocean freight industry: the carriers and the shippers.[1] A carrier is a logistics company who owns ocean liners for transporting huge

[1] While a freight forwarder can be viewed as a major player who serves as the "wholesaler" of the shipping capacities provided by the carriers, we neglect this entity to simplify our exposition.

number of TEU (Twenty-foot Equivalent Unit) containers by sea. For example, Maersk is the largest carrier in the world with a total annual capacity of 2,587,820 TEU containers, while OOCL is the largest carrier in Hong Kong with a total annual capacity of 468,079 TEU containers according to Alphaliner's list of top 100 carriers in 2013.[2] Also, shippers (customers) comprise of different types of supply chain partners – suppliers such as YKK (zippers), contract manufacturers such as Foxconn (iPhones), OEM manufacturers such as Philips (TVs) – who need to ship their products in TEU containers through ocean carriers.

Unlike the commercial airline industry, the price of ocean freight is based on a very special system (also known as the "Conference" system) that allows carriers being exempted from the Anti-trust laws. Specifically, carriers who serve the same route can jointly set the shipping schedules and prices to share their shipping capacities. Due to concern about price collusion, the European Commission and the European Liner Affair Association (ELAA) had decided that all carriers who ship from or to Europe have to abolish the conference system effectively on October 18, 2008. This has triggered a sea change in the ocean freight industry – the price is now set by individual carriers. Without a well-established pricing system for the ocean freight industry, the price of ocean freight has become more volatile since 2009. Besides demand seasonality,[3] the ocean freight price volatility is exacerbated further by three major factors: price volatility of crude oil, trade imbalance among different continents, and demand volatility.[4] Because ocean freight is viewed as indirect cost incurred as part of the supply chain operations, shippers are more concerned about huge price fluctuations even though they are obligated to ship their products.

Currently, there are two types of pricing contracts in the ocean freight logistics industry: (1) carriers sell their capacity (vessel space measured in terms of TEU containers) directly to shippers; or (2) carriers sell their capacity to shippers through intermediaries (i.e., freight

[2] Alphaliner - Top 100 (2013): http://www.alphaliner.com/top100/index.php

[3] In the ocean freight industry, peak season occurs between July and November to satisfy the demand in North America and Europe during Thanksgiving and Christmas holidays, while low season occurs between February and June.

[4] The freight rate during the peak season can be three times higher than that of the low season especially in two major routes (Transpacific, and Europe-Far East) with highly imbalanced trade. For example, more than half of containers flowing from Asia to North America (or Europe) return to Asia in empty container.

forwarders). In the former case, the carrier offers a fixed (regular) price according to a long-term (e.g., one year) contract and shippers who accepted this contract are known as Beneficial Container Owners (BCOs). In the latter case, the freight forwarders serve as "wholesale agents" who buy the space from different carriers and sell the consolidated capacity to shippers as a secondary market (spot market) on a short-term basis (one week prior to the departure date of the carrier). The "regular price" is known a prior in the former case; however, the "spot price" is uncertain ex-ante and is realized prior to the departure date. While over 70% of the transpacific ocean logistics trade are conducted directly between the carriers and the shippers, approximately 70% of the ocean logistics trade between Asia and Europe are contracted through freight forwarders in the spot market (Merge Global 2008).

The two aforementioned contracts types are fundamentally different. Under the long-term contract, the (regular) freight price is fixed and known in advance. For example, a BCO may negotiate the freight price with a carrier in March for the shipment that takes place from June until the following year. Clearly, the carrier prefers this long term (one-year) contract due to better planning and higher profit margins. However, some shippers are reluctant to accept a long term contract with a fixed and known freight price because they may be able to get a lower price if they postpone their booking decisions and buy the capacity from the freight forwarders (i.e., the spot market). This reluctance may help explaining why 70% of the shippers involving with trade between Asia and Europe prefer to buy space through freight forwarders in the spot market.

The shipper's interest in the spot market has caused major concerns for the carriers because their profit margins are lower if they sell their capacity to shippers through freight forwarders (as intermediaries) in the spot market. Furthermore, enticing shippers to sign contracts directly with the carrier will benefit the carrier for the early lock-in market share. Therefore, some major carriers in Asia are considering certain incentive schemes that are intended to entice shippers to buy the capacity directly from the carrier instead of the spot market. Specifically, based on our discussion with a major carrier in Hong Kong, we learned that this carrier is developing a "fractional" price matching contract that can be described as follows: the shipper pays a regular freight price in advance; however, the shipper will get a refund "only if" the realized spot price is below the regular price, where the refund is based on a "fraction" of the price difference (i.e., the difference between the regular price and the spot price). Clearly, if this fraction is equal to one, then this contract is the full price matching contract that is commonly observed in the retailing industry.[5]

By matching the spot price on a fractional basis, the carrier bears some of the risk associated with the uncertain spot price under the fractional price matching contract. This additional "risk" raises some interesting research questions: (1) What is the rational booking behavior of shippers under the fractional price matching contract? (2) How should the carrier set the regular price and the "fraction"? (3) Should the carrier offer the fractional price matching contract by bearing additional risk associated with the uncertain spot price? To our knowledge, there is no prior study of fractional price matching contracts. Even though full price matching contracts have been examined in the retailing industry, we need to develop a different model to capture the following characteristics of the ocean freight industry (that is inherently different from the retailing industry) including: (1) the shipper's demand is uncertain ex-ante; (2) the spot price is uncertain ex-ante and the realized spot price can be higher or lower than the regular price; and (3) the spot price is likely to be dependent on the spot market demand.

In Chapter 3, we present a model that captures the aforementioned characteristics of the ocean freight industry. Specifically, we model the dynamics between the carrier and the shippers as a Stackelberg game in which the carrier acts as the leader who sets the fractional price matching policy, and the shippers are the followers who decide whether to book with the carrier in advance. By anticipating the booking behavior of the shippers, we determine the optimal fractional pricing matching policy for the carrier. Our analysis enables us to draw the following conclusions. First, we show that the carrier can use the fractional price matching contract to generate a higher demand from the shippers by increasing the

[5]For example, Best Buy refunds the price difference if a customer finds a lower price elsewhere for the same product. See www.bestbuy.com for details.

"fraction" in equilibrium. Second, there are multiple optimal regular prices and the optimal "fraction". However, the optimal fractional price matching contract exhibits the following property: if the carrier offers a higher "fraction", then the carrier should increase the regular price to compensate for bearing additional risk. Third, we find that the optimal fractional price matching contract is "revenue neutral" in the sense that it enables the carrier to obtain the same expected revenue as before (when there was no price matching). This result implies that the carrier can develop a menu of fractional price matching contracts that are "revenue neutral" and let the shipper to choose a specific contract as desired. Finally, we show that our results continue to hold when certain assumptions are relaxed.

Overall, our study makes two contributions to the research literature. First, we contribute to the ocean freight literature even though pricing issues have been examined in the air and land logistics literature. Second, we contribute to the rich literature of price matching policy, which has been studied from various angles: economics, marketing, and operations management. Chapter 3 is organized as follows. We review relevant literature in Section 3.2. In Section 3.3, we first present a base model of the fractional price matching policy. Then, we examine the shipper's booking behavior. We determine the carrier's optimal fractional price matching policy and its implications in Section 3.4. Section 3.5 and 3.6 present two different extensions of the base model: general shipper demand model and dependent spot price model. This chapter ends with some concluding remarks in Section 3.7.

3.2 Literature Review

This study is related to two research streams: logistics management, and price matching. First, pricing issues in logistics industry have been studied extensively in air and land logistics (see for example, [ACG11] for air, [YBS95] for railroad transportation, [KVB04] and [KAS07] for tanker and dry bulk shipping). However, very little literature has addressed sea logistics. [WL95] provide a linear programming-based pricing model to assist shipper's contracting with ocean container shipping company. [LRX08] provide linear programming model to assist a global electronics provider to decide the bidding price in each segment of the shipping router. They incorporate the result from sensitivity analysis. [Gar07] studies the case in which shipping demand is elastic to the transportation service prices. In his model, both bidding price and carrier's empty container reposition cost are considered. He uses the double-auction scheme to help the shippers to decide when to buy the transportation service (and how much) to minimize total inventory and transportation costs, while we will use a game-based approach to develop a win-win situation for both carrier and shippers.

Underlying our study is also the research stream in ***economics*** that investigates the impact of price matching policies on the market in a competitive environment. At a first glance, price-matching policies would appear to be pro-competitive. However, the dominant view in the economics literature is that price matching is anti-competitive as it can facilitate monopoly pricing ([Sal86, Bel87, Doy88, EE99]). Using static oligopoly models, they show that price matching policies can support monopoly pricing (i.e., cartel pricing). Specifically, when all firms offer price matching policies, there is no incentive to undercut the prices of rivals. Reduced competition then results in high prices, indeed monopolistic ones. Another line of research argues that price matching is a form of price discrimination based on customer's price knowledge ([Bel87, PH87, Edl97]). They show that by offering no match its competitor's price, a firm gives discounts to customers who are aware of the market prices but it keeps the price to other customers. In sum, economics predominantly view price matching as an anti-competitive practice due to monopoly pricing and price discrimination. However, one can still observe price matching policies in practice (e.g., retail, hotel and airline industry, etc.). Our research contributes to this literature by explaining why firms keep offering price matching policies even though they are shown to be anti-competitive, and analyzing the benefits of price matching policies, specifically in ocean freight industry.

Our study is related to the literature which investigates different variants of price matching policy. The format of the fractional price matching where the firm can control the **refund-depth** has been studied recently ([Tum11, HS12, DLV12]).[6] [Tum11] considers two

[6]By controlling the refund-depth, the firm can refund a fraction of price difference. Hence, controlling

identical firms competing in two-stage: In the first stage, each firm installs its capacity which is the maximal quantity that the firm can sell in the second stage. In the second stage, after observing the other firm's capacity, each firm chooses its announced price and price matching option, where the price matching option follows the format of the fractional price matching. [HS12] consider a situation in which price matching guarantees are offered by small local firms who compete against much larger rivals. They investigate small firm's incentives to offer price matching guarantees in such environments by adopting the format of the fractional price matching in their model. Using an experimental study, [DLV12] examine the refund depth effects on price-matching guarantees. They show that if a store has high price image, deep price guarantee is not an optimal strategy to retain regular customers. Even though this stream of research considers the format of the fractional price matching policy, our study can contribute to this literature differently because they focus on the "horizontal competition" among multiple horizontal firms while our study focuses on "vertical competition" of the carrier and the shipper where the carrier offers the price matching policy to the shipper to encourage him reserving more with her.

In ***operations*** and ***marketing*** area, scholars study the "full" price matching policies with considering many different aspects of industry (e.g., retail assortment decisions ([CS03]), verification of product availability ([NBR10]), consumer search behavior ([CNZ01]), consumer's strategic purchasing behavior ([LD10])). [CS03] consider retail assortment decisions of retailers who offer "full" price matching policies, and prove that if the shelf space is limited, then retailers carry non-overlapping product lines, and the equilibrium prices depend on the degree of substitutability. [NBR10] analyze the role of verification of product availability in the context of competitive price matching policies. They show that the verification of availability can result in customers paying lower retail prices by increasing the level of retail price competition. By considering consumer behavior with presence of price matching policies, [JS00, SL04, MZ06] show that price-matching can signal low prices and low service levels of stores to the customers. If the existence of price-matching causes consumer search

the refund depth is a similar concept with the format of the fractional price matching policy.

behavior to increase, [CNZ01] show that price matching policies can worsen seller's profits if there are different segments of consumers with different search costs. [LD10] examine the impact of price matching policy on customer's purchasing behavior, and seller's pricing and inventory decisions. They find that the price matching policy eliminates strategic customer's waiting incentive, and thus allows the seller to increase price in the regular season. More closely related to operations management area, [LMN07] investigate a revenue-management problem in which a monopolistic seller can sell the product along with price matching. By formulating the problem as a discrete-time dynamic program, they characterize the optimal decisions on the price path and the price matching policy. Even though many scholars have visited different industries and analyzed the impact of "full" price matching policies on them, ocean freight industry has not been tackled yet especially in the presence of "fractional" price matching. By considering the characteristics of ocean freight industry, our study can contribute to this literature by providing different managerial insights about this untapped industry.

3.3 Fractional Price Matching Policies

Consider an ocean freight carrier who needs to manage the revenue of its limited capacity K over two time periods. In the first period, the carrier sells its capacity to potential shippers (customers) at a unit price r. At the beginning of the second period, the spot market opens at price s per unit, where s is uncertain ex-ante in the first period but it is realized at the beginning of the second period. In the second period, the carrier sells its remaining capacity on the spot market at θs per unit, where $\theta \in (0, 1)$ represents the discount that captures the transaction cost and other handling fees. Unlike commercial airline industry that serves consumers, the B2B ocean freight carrier does not overbook by accepting reservations for more than K units because of the following reasons: 1) all bookings conducted in the first period are confirmed; and 2) all bookings are not cancellable (i.e., shippers cannot sell the unneeded containers back to the carrier).

There are $N(> K)$ shippers in the market. Shipper $i \in \{1, 2, ..., N\}$ faces uncertain demand D_i, where D_i is uncertain ex-ante in the first period but it is realized at the beginning of the second period for all i. Specifically, $D_i = 1$ with probability ρ_i, and $D_i = 0$ with probability $1 - \rho_i$

Given the demand and spot price uncertainties, each of the N shippers needs to evaluate the tradeoffs between reserving from the carrier before the uncertainties are resolved and buying from the spot market after all uncertainties are resolved. If the shipper books 1 unit of the capacity with the carrier in the first period and then discovers no shipment is needed (because $D_i = 0$) in the second period, then he can sell the reserved unit on the secondary market at θs, where $\theta \in (0, 1)$ represents the discount that captures the transaction cost and other handling fees.[7]

To encourage shippers to book with the carrier in the first period instead of buying from the spot market in the second period, we consider the "fractional price matching" policy that is under consideration by a carrier in Hong Kong. Under the fractional price matching policy (r, β), the effective unit price that a shipper has to pay for any realized spot price s is given by:

$$m(r, \beta, s) \quad = r - \beta[r - s]^+ = \quad r - \beta(r - \min\{r, s\}), \tag{3.1}$$

where $\beta \in [0, 1]$. Specifically, the shipper pays the original unit price r if the realized spot price $s \geq r$, and pays $(1 - \beta)r + \beta s$ (i.e., a "linear convex" combination of the original price r and the realized spot price s) if $s < r$. Notice that the carrier offers "full price matching" when $\beta = 1$ and "no matching" when $\beta = 0$. Hence, a shipper's effective unit price is $m(r, \beta, s)$ if he books with the carrier in period 1. If the shipper does not book in period 1, we assume that he can always get the capacity to ship by paying the spot price s, where s is uncertain ex-ante with $Exp(s) \equiv \mu$.

Besides the fractional price matching policy, another carrier in Hong Kong is considering a slightly different price matching policy ("partial price matching policy") under which the

[7]While we assume the "discount" factor to be the same for the carrier and the shipper for ease of exposition, one can obtain the same result even when this "discount" factor is firm-specific.

shipper pays the regular price r if the realized spot price s in period 2 is "not too low" (i.e., when $s \geq r - \alpha$) and the shipper pays the realized spot price s if $s < r - \alpha$, where $\alpha \in [0, r]$ is the matching parameter. In this case, the carrier offers "full matching" when $\alpha = 0$, "no matching" when $\alpha = r$, and "partial matching" when $\alpha \in (0, r)$. As it turns out, it can be shown that there is a one-to-one correspondence between the partial matching policy and the fractional matching policy so that one can retrieve the optimal partial price matching policy from the optimal fractional price matching policies (r^*, β^*). We shall explain this in more details in Appendix 1. To avoid repetition, we shall focus our analysis on the fractional pricing matching policy (r, β).

To determine the optimal fractional price matching policies (r^*, β^*), we shall model the dynamics between the carrier and the shippers as a Stackelberg game in which the carrier acts as the leader who sets the fractional price matching policy (r, β), and the shippers are the followers who decide whether to book with the carrier in period 1 or not. To begin, we examine the shipper's booking behavior.

3.3.1 The carrier's pricing behavior and the shipper's booking behavior

For notational convenience, let $E = Exp\{m(r, \beta, s)\}$ denote the "expected" effective price under the fractional price matching policy, where E depends on the fractional price matching policy (r, β) and the distribution of s. By noting that all shippers will buy on the spot market if $\mu < E$ and that the carrier will not sell to the shippers (and sell on the spot market instead) if $E < \theta\mu$, we can ignore these trivial cases by focusing on the case when the expected effective price E satisfies:

$$\theta\mu \leq E \leq \mu. \tag{3.2}$$

Now, let us examine the tradeoff that a shipper faces. First, if shipper i books with the carrier in period 1, then his net expected cost is equal to $E - (1 - \rho_i)\theta\mu$, where the first term is the expected cost of the booking in period 1 under the fractional price matching policy and the second term is the expected "salvage value" to be obtained from selling the unneeded reservation on the spot market after discovering that the reservation is not needed in period 2. Second, if shipper i buys on the spot market in period 2 in the event that the shipment is needed, the expected cost is $\rho_i \mu$. By comparing the costs associated with these two options, it is easy to check that shipper i will book with the carrier in period 1 if ρ_i satisfies $\rho_i \geq \frac{E-\theta\mu}{\mu(1-\theta)}$. While ρ_i is private information of the shipper, the carrier only knows that the probability ρ_i of all N shippers is distributed according to a distribution, which is common knowledge. Therefore, the proportion of shippers who will book in advance with the carrier is given by:

$$q(r, \beta) = Prob[\rho \geq \frac{E - \theta\mu}{\mu(1-\theta)}]. \tag{3.3}$$

Observe from (3.1) that $E \leq r$, E is non-decreasing in r , and that E is decreasing in β. Consequently, by considering (3.3), it is easy to show that:

Corollary 1. *Under the fractional price matching policy (r, β), the carrier can generate a higher demand (via a higher value of $q(r, \beta)$) with a lower expected effective price E by increasing the "fractional price matching" parameter β.*

3.3.2 The carrier's problem

By anticipating each shipper's rational booking behavior, we now formulate the carrier's problem. First, for any given fractional price matching policy (r, β), the expected booking quantity generated by the shippers in period 1 is given by $N \cdot q(r, \beta)$. Hence, the carrier's expected revenue is equal to:

$$\Pi(r, \beta) \quad = \quad E \cdot \min\{N \cdot q(r, \beta), K\} + \theta\mu \cdot [K - N \cdot q(r, \beta)]^+, \tag{3.4}$$

where the first term is the revenue obtained from the shipper in period 1 and the second term is the revenue obtained from selling the remaining capacity on the spot market in period 2. The carrier's problem can be formulated as:

$$\max_{r,\beta} \;\; \Pi(r, \beta) \;\; s.t., \quad (3.2). \tag{3.5}$$

3.4 Optimal Fractional Price Matching

We now determine the optimal fractional price matching policy (r^*, β^*) by solving the carrier's problem defined in (3.5). For tractability and for ease of exposition, we consider the case when the demand probability $\rho \sim U[0,1]$ and the spot price $s \sim U[0,1]$, where ρ and s are independent (We shall examine the case when the spot price s depends on market condition in a later section). In this case, the average spot price is $\mu = 0.5$. Also, it is easy to check from (3.1) and (3.3) that

$$E = r - \beta\frac{r^2}{2}, \quad \text{and} \tag{3.6}$$

$$q(r,\beta) = 1 - \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}. \tag{3.7}$$

Notice that the expected effective price E is increasing and concave in the regular price r for $0 \le r \le \frac{1}{\beta}$, and the proportion of shippers who will book from the carrier $q(r,\beta)$ is decreasing in the expected effective price E. These two properties are intuitive.

By using (3.6), (3.7) and the fact that $\mu = 0.5$, the carrier's problem defined in (3.5) can be rewritten as the following mathematical program that has E as a decision variable:

$$\max_E \quad E \cdot \min\{N \cdot (1 - \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}), K\} + \theta\mu \cdot [K - N \cdot (1 - \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)})]^+, \tag{3.8}$$
$$\text{subject to} \quad 0.5\theta \le E \le 0.5.$$

Lemma 1. *Under the fractional price matching policy, there exists an optimal solution E^* to program (3.8) that has $N \cdot (1 - \frac{E^* - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}) \le K$.*

Proof All formal proofs are provided in Appendix 3.2.

In view of Lemma 1, we can simplify program (3.8) as:

$$\max_E \quad E \cdot N \cdot (1 - \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}) + \theta\mu \cdot (K - N \cdot (1 - \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)})), \tag{3.9}$$
$$\text{subject to} \quad 0.5\theta \le E \le 0.5, \tag{3.10}$$
$$N \cdot (1 - \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}) \le K. \tag{3.11}$$

By considering the objective function (3.9) along with the bounds, one can show that:

Proposition 1 *Under the fractional price matching policy* (r,β) *as defined in (3.1), the optimal effective price* E^* *associated with the optimal fractional price policy* (r^*,β^*) *satisfies:*

$$E^* \quad = \quad \frac{1}{2}[\theta + (1-\theta)\max\{0.5, 1-\frac{K}{N}\}]. \tag{3.12}$$

Also, the optimal fractional price policy (r^*,β^*) *satisfies:*

$$r^* - \beta^*\frac{(r^*)^2}{2} \quad = \quad E^* = \frac{1}{2}[\theta + (1-\theta)\max\{0.5, 1-\frac{K}{N}\}]. \tag{3.13}$$

Proposition 1 has the following implications. First, under the optimal fractional price matching policy (r^*,β^*), the corresponding optimal expected effective price E^* given in (3.12) is decreasing in her capacity K. Second, it follows from (3.13) that the optimal fractional price matching policy (r^*,β^*) is not unique. For example, when the carrier offers no price matching so that $\beta^*=0$, the corresponding optimal regular price $r_0^* = \frac{1}{2}[\theta + (1-\theta)\max\{0.5, 1-\frac{K}{N}\}]$, where the term $\frac{\theta}{2}$ is the carrier's expected salvage value in the spot market in period 2, and the term $\frac{1-\theta}{2}\max\{\frac{1}{2}, 1-\frac{K}{N}\}$ is the "additional revenue" for the carrier to obtain by selling her capacity in period 1. Next, when $\beta^* \in (0,1]$, the corresponding optimal regular price $r_\beta^* = \frac{1}{\beta^*}[1-\sqrt{1-\beta^*\{(1-\theta)\max\{0.5, 1-\frac{K}{N}\}+\theta\}}$, where r_β^* is increasing in β^*. This result is intuitive because, when the carrier offers a contract with a higher price matching parameter β^*, the carrier bears additional risk associated with the uncertain spot market. Therefore, in order to compensate for bearing this additional risk, the carrier should charge a higher regular price r_β^*.

Finally, observe from (3.13) that all optimal fractional price matching policies (r^*,β^*) yield the same expected effective price $E^* = \frac{1}{2}[\theta + (1-\theta)\max\{0.5, 1-\frac{K}{N}\}]$. Consequently, it is easy to check from (3.8) that all optimal fractional price matching policies (r^*,β^*) generate the same expected revenue for the carrier. This result implies that the optimal fractional price matching contract is "revenue neutral" in the sense that it enables the carrier to obtain the same expected revenue as before (when there was no price matching). Therefore, under the optimal fractional price matching policies (r^*,β^*), the carrier can offer a "menu" of regular price r^*_β for each price matching parameter β^* and obtain the same expected revenue. This way, the carrier can let the shipper to choose the fractional price matching contract according to their preference.

3.4.1 Optimal Fractional Price Matching: when the regular price r is given

The optimal fractional price matching policies (r^*, β^*) given in (3.13) are based on the assumption that the regular price r is a decision variable. However, in many instances, the long term regular price r may be set way in advance. In the event when r is given, we now examine the property of the optimal price matching parameter $\beta^*(r)$ as a function of r. To eliminate the trivial cases, let us consider the case when the given regular price r satisfies (3.2) so that $0.5\theta \leq r \leq 0.5$.

By using the fact that $0.5 \geq r$, it is easy to check from (3.12) that the optimal effective price $E^* = \frac{1}{2}[\theta + (1-\theta)\max\{0.5, 1 - \frac{K}{N}\}] \geq \frac{1}{2}[0.5] \geq \frac{1}{2}r$. Also, by using (3.13), it is easy to check that the optimal price matching parameter $\beta^*(r) \in [0,1]$ for any given r is equal to:

$$\beta^*(r) \quad = \quad \max[0,\ \min\{1, \frac{r - E^*}{r^2/2}\}].$$

We can use the expression for $\beta^*(r)$ to make three observations. First, by using the fact that $E^* \geq \frac{1}{2}r$, it is easy to check that the optimal price matching parameter $\beta^*(r)$ is weakly increasing in r. Hence, when the given regular price r is high, it is optimal for the carrier to increase her optimal price matching parameter $\beta^*(r)$. This result is intuitive. Second, by using the fact that $E^* = \frac{1}{2}[\theta + (1-\theta)\max\{0.5, 1 - \frac{K}{N}\}]$, it is easy to check that $\beta^*(r)$ is weakly increasing in $\frac{K}{N}$. Hence, when the carrier's capacity K is high, it is optimal for the carrier to increase her price matching so as to entice more shippers to book with her in period 1. Third, by considering E^*, it is easy to check that $\beta^*(r) = 1$ when θ is small (i.e., $\theta \leq 1 - \frac{(r-1)^2}{1-\max\{0.5,1-K/N\}}$), then $\beta^*(r)$ is decreasing in θ in the medium range (i.e., $1 - \frac{(r-1)^2}{1-\max\{0.5,1-K/N\}} < \theta < \frac{2r-\max\{0.5,1-K/N\}}{1-\max\{0.5,1-K/N\}}$, and then $\beta^*(r) = 0$ when θ is large (i.e., $\theta \geq \frac{2r-\max\{0.5,1-K/N\}}{1-\max\{0.5,1-K/N\}}$). Hence, when the discount factor in selling the unneeded capacity in the spot market increases (i.e.,θ increases), the carrier can afford to reduce her price matching $\beta^*(r)$ because the shippers are more eager to book with the carrier because they can obtain higher salvage value on the spot market in the event when their reserved capacity is not needed.

3.4.2 Numerical results for the case when the spot price s is Normally distributed

Instead of assuming the spot price s is uniformly distributed, we now consider the case when $s \sim Normal(\mu, \sigma^2)$. However, there is no closed form expression for the optimal price matching policy. Instead, we solve the mathematical program (3.5) numerically. In our numerical study, we set $N = 1000$, $K = 500$, $\theta = 0.8$, and we are interested in comparing three optimal policies: (1) Optimal policy when there is no price matching – the optimal regular price is denoted by r_B^*; (2) Optimal fractional price matching policy – the optimal policy is denoted by (r_β^*, β^*); and (3) Optimal partial price matching policy[8] – the optimal policy is denoted by (r_α^*, α^*). Also, for each optimal policy, we compute the corresponding optimal expected effective price and optimal revenue; namely, E_B^*, E_β^*, E_α^* and Π_B^*, Π_β^*, Π_α^*; respectively.

To begin, we investigate the impact of the average spot price (μ) on the optimal policies. To do so, we fix $\sigma = 300$, and vary μ from 600 to 1400 using increments of 200. Our result is summarized in Table 3.1. Table 3.1 has the following implications that are intuitive and consistent with our analytical results for the case when $s\ U[0,1]$: (1) when the average spot price μ increases, the carrier can afford to increase her optimal effective prices (E_B^*, E_β^*, E_α^*) associated with all three optimal policies; (2) as the carrier bears some of the risk associated with uncertain spot price under price matching, the optimal regular price under price matching is higher than the regular price with no price matching (i.e., $r_B^* < r_\beta^*,\ r_\alpha^*$). Notice that this result is consistent with the result as stated in Proposition 1; and (3) the optimal expected effective prices (E_B^*, E_β^*, E_α^*) and the optimal revenue (Π_B^*, Π_β^*, Π_α^*) associated with

[8] While we show analytically that the optimal partial price matching policy is equivalent to the optimal fractional price matching policy in Appendix 3.1 for the case when $s\ U[0,1]$, it is unclear if this result holds when $s\ Normal(\mu, \sigma^2)$

all three optimal policies are identical, and increasing in μ. Hence, this numerical result for the case when $s \sim \; Normal(\mu, \sigma^2)$ is consistent with our analytical result for the case when $s \; U[0, 1]$.

Next, we investigate the impact of the standard deviation of the spot price (σ) on all three optimal policies. To do so, we fix $\mu = 1000$, and vary σ from 300 to 600 using increments of 100. Our result is summarized in Table 3.2. Table 3.2 reveals some interesting observations: (1) the optimal expected effective prices (E^*_B, E^*_β, E^*_α) associated with all three optimal policies are not affected by the spot price uncertainty captured by σ;[9] (2) To compensate for the risk associated with uncertain spot price that the carrier is bearing under price matching, the optimal regular price under price matching is higher than the regular price with no price matching (i.e., $r^*_B < r^*_\beta, \; r^*_\alpha$); (3) To compensate for the risk associated with uncertain spot price that the carrier is bearing under price matching, it is optimal for the carrier to decrease her regular price r^*_β, and reduce price matching (by decreasing β^*) as spot price uncertainty σ increases, and keep the optimal effective price E^*_β constant; (4) The optimal partial price matching policy suggests that it is optimal for the carrier to increase her regular price r^*_α, and reduce price matching (by increasing α^*) as spot price uncertainty σ increases, and keep the optimal effective price E^*_α constant.

By noting that the optimal expected effective prices (E^*_B, E^*_β, E^*_α) associated with all three optimal policies are independent of spot price uncertainty σ, it is no surprise that the carrier's optimal expected revenue (Π^*_B, Π^*_β, Π^*_α) associated with all three optimal policies are also independent spot price uncertainty σ. Therefore, even though the optimal fractional price matching policy (r^*_β, β^*) is affected by σ, we show that the optimal fractional price matching policy (r^*_β, β^*) enables the carrier to reduce the impact associated with the additional risk caused by price matching to zero. In other words, the optimal fractional price matching policy (r^*_β, β^*) is "revenue neutral" in the sense that it enables the carrier to obtain the same expected revenue as before (when there was no price matching) even when

[9] When there is no price matching (i.e., $\beta = 0$), observe from (3.1) that the effective expected price E and the corresponding carrier's problem (3.5) are independent of σ. This explains why the optimal regular price r^*_B, the optimal expected effective price E^*_B, and the optimal revenue Π^*_B are independent of σ.

the carrier bears some of the risk associated with the uncertain spot price.

3.5 Extension 1: General Shipper Demand

In the base model, we assume that the uncertain demand of each shipper is binary so that his booking decision in period 1 is also binary. We now extend the base model to the case of a single shipper whose uncertain demand D follows a probability distribution over a certain range instead of 0 or 1 in the base case.[10] Let us first consider the shipper's problem. Under the fractional price matching policy (r, β) given in (3.1), the shipper's expected cost for booking x containers in period 1 for any realized demand d and spot price s can be expressed as:

$$\pi(x; r, \beta; s, d) \quad = \quad x \cdot m(r, \beta, s) + s \cdot [d - x]^+ - \theta s \cdot [x - d]^+, \tag{3.14}$$

where the first term is the effective booking cost in period 1, the second term is cost for booking additional units from the spot market in period 2 when there is a shortage, and the third term is the "salvage value" for selling those excessive units on the spot market in period 2. By taking the expectation of the shipper's cost with respect to the demand d and spot price s, and by taking the derivative of the expected cost with respect to the booking quantity x in period 1, it is easy to obtain the following result by examining the first-order condition:

Lemma 2 *Under the Fractional Price Matching Policy (r, β) given in (3.1), it is optimal for the shipper to book x^* containers in period 1, where x^* satisfies:*

$$Prob\{D > x^*\} \quad = \quad \frac{E - \theta\mu}{\mu(1 - \theta)}. \tag{3.15}$$

[10]The multi-shipper case is beyond the scope of our study because the analysis becomes intractable. We shall defer it as future research

It follows from (3.2) that the quantity $\frac{E-\theta\mu}{\mu(1-\theta)} \in [0,1]$. Suppose we interpret (3.15) as the proportion of the demand ($Prob\{1 > \frac{x^*}{D}\}$) that the shipper should book with the carrier in period 1. Then this interpretation is similar to proportion of the shippers who books with the carrier in period 1 as defined in (3.3) in the base case. Hence, this observation explains why the right hand sides of both equations are identical.

3.5.1 The carrier's problem

Anticipating the shipper's optimal booking quantity x^* that satisfies (3.15), we now formulate the carrier's problem by noting that the carrier's expected revenue under the fractional price matching policy (r,β) is equal to: $\Pi(r,\beta) = E \cdot \min\{x^*, K\} + \theta\mu \cdot [K - x^*]^+$, where the first term is the revenue obtained from the shipper in period 1, and the second term is the revenue obtained from selling the remaining capacity on the spot market in period 2. In this case, the carrier's problem can be formulated as: $\max_{r,\beta} \Pi(r,\beta)$, subject to (3.2).

To obtain tractable result, let us consider the case when $D \sim U[m-\eta, m+\eta]$ and the spot price $s \sim U[0,1]$, where D and s are independent (our analysis can be easily extended to the case when s is uniformly distributed over any specific range). In this case, the average spot price is $\mu = 0.5$. Also, it is easy to check from (3.1) and (3.15) that:

$$\begin{aligned} E &= \quad r - \beta\frac{r^2}{2} \; and \\ x^* &= \; (m+\eta) - 2\eta(\frac{E-0.5\theta}{0.5(1-\theta)}) \end{aligned} \tag{3.16}$$

By using the same argument as presented in Lemma 1, the carrier's problem can be formulated as a program that has E as the decision variable: $\max_E E \cdot x^* + 0.5\theta \cdot [K - x^*]$ subject to $0.5\theta \le E \le 0.5$, and $x^* \le K$, where x^* is a function of E given in (3.16). By considering the first order condition (in terms of E), and by considering the bounds on E, one can show that:

Proposition 2. *Under the fractional price matching policy (r,β) as defined in (3.1), the* *optimal effective price E^* associated with the optimal fractional price policy (r^*, β^*) satisfies:*

$$E^* \quad = \quad \max[\frac{(m+\eta-K)}{4\eta}(1-\theta)+0.5\theta\,,\ \min\{0.5,\ \frac{(m+\eta)}{8\eta}(1-\theta)+0.5\theta\}]. \quad (3.17)$$

Also, the optimal fractional price policy (r^*, β^*) satisfies:

$$r^* - \beta^* \frac{(r^*)^2}{2} \quad = \quad E^*. \quad (3.18)$$

Proposition 2 is akin to Proposition 1, and it has the following implications. First, under the optimal fractional price matching policy (r^*, β^*), the optimal expected effective price E^* given in (3.17) is non-increasing in her capacity K. This result is intuitive. Second, it follows from (3.18) that the optimal fractional price matching policy (r^*, β^*) is not unique. Also, it is easy to check from (3.18) that the optimal regular price r^* is increasing in β^*: when the carrier bears more risk caused by the spot market by offering a higher price matching β^*, the carrier can compensate for this additional risk by charging a higher regular price r^*. Finally, observe from (3.18) that all optimal fractional price matching policies (r^*, β^*) yield the same expected effective price E^*. Consequently, it is easy to check from that all optimal fractional price matching policies (r^*, β^*) generate the same expected revenue for the carrier. Therefore, our result for the base case continues to hold when the shipper's demand D follows a probability distribution over a certain range (instead of 0 or 1 in the base case).

3.6 Extension 2: Dependent Spot Price

In the base model, we assume that the spot price s is independent of the shipper's demand. In this section, we deal with the case when the spot price s depends on the expected demand in period 2. Recall from Section 3.1 that shippers who did not book in period 1 must have demand probability $\rho \leq \frac{E-\theta\mu}{\mu(1-\theta)}$. Hence, by using the fact $\rho \sim U[0,1]$, the expected demand in period 2 is equal to $N \cdot \frac{1}{2} \cdot \frac{E-\theta\mu}{\mu(1-\theta)} = N\frac{1-q(r,\beta)}{2}$, where $q(r, \beta)$ is given in (3.3). To model the dependency between the spot price s and the expected demand in period 2, we consider the case when

$$s \quad = \quad \tau \cdot N\frac{1-q(r,\beta)}{2} + \epsilon, \tag{3.19}$$

where $\epsilon \sim U[0,1]$ and $\tau \geq 0$ (notice that this extension reduces to the base case when $\tau = 0$). While the spot price s depends on the expected demand in period 2, the demand in period 2 depends on the shipper's booking behavior in period 1 via $q(r,\beta)$, which depends on the spot price s. Hence, this "circular" relationship implies that the expected spot price $\mu = E(s)$ should be determined "endogenously."

3.6.1 Special case: when $\beta = 0$.

To simplify our exposition, let us consider the case when the carrier offers no price matching with $\beta = 0$. It follows from (3.1) that $m(r,\beta,s) = r$ when $\beta = 0$. By using the fact that the expected effective price $E = r$, it is easy to check from (3.3) and (3.19) that the proportion of shippers who will book with the carrier in period 1 (i.e., $q(r)$) and the expected spot price μ satisfy:

$$q(r) \quad = \quad 1 - \frac{E-\theta\mu}{(1-\theta)\mu} = 1 - \frac{r-\theta\mu}{(1-\theta)\mu}, \text{ where} \tag{3.20}$$

$$\mu = Exp(s) \quad = \quad \frac{1}{2}\tau N(\frac{r-\theta\mu}{(1-\theta)\mu}) + 0.5. \tag{3.21}$$

It follows from (3.21), we can determine the expected spot price $\mu(r)$ "endogenously" by solving a quadratic equation, where:

$$\mu(r) = \frac{-(\tau N\theta - (1-\theta)) + \sqrt{(\tau N\theta - (1-\theta))^2 + 8\tau N r(1-\theta)}}{4(1-\theta)} > 0. \tag{3.22}$$

By using the same argument as stated in Lemma 1, it can be shown that the carrier's problem can be formulated as:

$$\max_r \qquad r \cdot N \cdot q(r) + \theta\mu(r) \cdot [K - N \cdot q(r)]$$

$$\text{subject to} \qquad \theta\mu(r) \le r \le \mu(r), \quad \text{and}$$
$$N \cdot q(r) \le K,$$

where $q(r)$ is given in (3.20), and $\mu(r)$ is given in (3.22). Note that the above program is a mathematical program with non-linear objective function and non-linear constraints associated with the decision variable r. As such, there is no closed form expression for the optimal regular price r^*. However, one can always determine the optimal r^* numerically. We shall present the numerical results in the next section where we discuss about the general case of this model (i.e., $\beta \in [0, 1]$).

3.6.2 General case: when $\beta \in [0, 1]$

We now consider the case when $\beta \in [0, 1]$. It follows from (3.3) and (3.19) that the proportion of shippers who will book with the carrier in period 1 (i.e., $q(r, \beta)$) and the expected spot price μ satisfy:

$$\begin{aligned} q(r,\beta) &= 1 - \frac{E - \theta\mu}{(1-\theta)\mu}, \text{ where} \\ \mu = Exp(s) &= \frac{1}{2}\tau N(1 - q(r,\beta)) + 0.5. \end{aligned} \qquad (3.23)$$

By noting from (3.19) that s is uniformly distributed between $\frac{1}{2}\tau N(1-q(r,\beta))$ and $\{\frac{1}{2}\tau N(1-q(r,\beta)) + 1\}$, we can use (3.1) to show that the expected effective price E satisfies:

$$\begin{aligned} E = Exp\{m(r,\beta,s)\} &= r - \beta \int_{\frac{1}{2}\tau N(1-q(r,\beta))}^{r} (r-s)ds \\ &= r - \beta\frac{r^2}{2} + \beta\{\frac{1}{2}\tau N(1-q(r,\beta))\}[r - \frac{1}{4}\tau N(1-q(r,\beta))] \end{aligned} \qquad (3.24)$$

By using $q(r, \beta)$ and μ given above, we can express the expected effective price E in terms of $q(r, \beta)$ so that $E = \{\frac{1}{2}\tau N(1 - q(r, \beta)) + 0.5\}[\theta + (1 - \theta)(1 - q(r, \beta))]$. Combine this observation with (3.24), we can determine the proportion of shippers who will book with the carrier in period 1 (i.e., $q(r, \beta)$) endogenously by solving the following quadratic equation of $q(r, \beta)$:

$$\{\frac{1}{2}\tau N(1 - q(r,\beta)) + 0.5\}[\theta + (1-\theta)(1 - q(r,\beta))] = \qquad (3.25)$$

$$r - \beta\frac{r^2}{2} + \beta\{\frac{1}{2}\tau N(1-q(r,\beta))\}[r - \frac{1}{4}\tau N(1-q(r,\beta))]$$

By solving (3.25), we show that $q(r,\beta)$ satisfies:

$$q(r,\beta) = \quad 1 - [\frac{\{\beta\tau Nr - \theta\tau N - (1-\theta)\} + \sqrt{\{\beta\tau Nr - \theta\tau N - (1-\theta)\}^2 - 4(\theta - 2r + \beta r^2)\{\tau N(1-\theta) + \frac{1}{4}\beta(\tau N)^2\}}}{2\{\tau N(1-\theta) + \frac{1}{4}\beta(\tau N)^2\}}] \quad (3.26)$$

By using $q(r,\beta)$ given in (3.26), we can determine the expected spot price $\mu(r,\beta)$ from (3.23), and use (3.24) to retrieve the expected effective price E. Again, by using the same argument as stated in Lemma 1, it can be shown that the carrier's problem can be formulated as:

$$\begin{aligned}
\max_{r,\beta} \quad & E \cdot N \cdot q(r,\beta) + \theta \cdot \mu(r,\beta) \cdot [K - N \cdot q(r,\beta))], \\
\text{subject to} \quad & \theta \cdot \mu(r,\beta) \leq E(r,\beta) \leq \mu(r,\beta), \quad \text{and} \\
& N \cdot q(r,\beta) \leq K,
\end{aligned}$$

where $q(r,\beta)$ is given above, and $\mu(r,\beta)$ can be computed from (3.23). It is easy to recognize that the carrier's problem is a mathematical program with non-linear objective function and non-linear constraints associated with the decision variables r and β. As such, there is no closed form expression for the optimal fractional price matching policy (r^*, β^*). Hence, we solve this mathematical program numerically to generate further insight. We use the same parameter setting given in Section 3.4.2 so that we can compare the results. Moreover, from the numerical analysis, we investigate how the spot price coefficient τ affects the optimal policy and the optimal revenue.

To begin, we consider the case when ϵ follows $Normal(\mu_\epsilon, \sigma_\epsilon^2)$ and we investigate the impact of ($\mu_\epsilon = Exp(\epsilon)$) on the optimal policies. To do so, we fix the standard deviation of ϵ by setting $\sigma_\epsilon = 300$ and vary μ_ϵ from 600 to 1400 using increments of 200. Our numerical result is summarized in Table 3.3. From Table 3.3, we observe the same pattern as in Table 3.1. Thus, Table 3.3 can be interpreted in the same way as Table 3.1. To avoid repetition, we omit the details. However, by examining the proportion of the shippers who book in period 1 (q^*) and the expected spot price (μ^*), we can draw additional conclusions. First, under all three policies (no price matching, fractional price matching, and partial price matching policies), the proportion of the shippers who book in period 1 (q^*) is increasing in μ_ϵ. This result is due to the fact that the "effective" expected spot price (μ^*) is increasing in μ_ϵ so that more shippers will book with the carrier in period 1 when the expected spot price in period 2 is increasing. Second, one can expect from (3.19) that an increase in q would result in an decrease in the expected spot price. However, Table 3.3 shows the opposite (i.e., the expected spot price increases). This result can be explained by the fact that the effect caused by the increase in μ_ϵ dominates the effect associated with q^*.

Next, we investigate the impact of the standard deviation (σ_ϵ) on the optimal policies. To do so, we fix $\mu_\epsilon = 1000$, and vary σ_ϵ from 300 to 600 using increments of 100. Our result is summarized in Table 3.4. From Table 3.4, we observe the same pattern as in Table 3.2. Hence, our intuition from Table 3.2 continues to hold when the spot price is dependent on the expected demand in period 2. We omit the details. However, it is interesting to see that all optimal policies yield the same expected effective prices, which generate the same expected revenue. Thus, our analytical and numerical results for the base case presented in the previous sections continue to hold even if we introduce the spot price dependency on the demand.

Finally, we investigate the impact of the spot price coefficient τ on the optimal policies. To do so, we fix $\mu_\epsilon = 1000$ and $\sigma_\epsilon = 300$, and vary τ from 0 to 1.5 using increments of 0.5. Our result is summarized in Table 3.5, which has the following implications. First, observe that the case $\tau = 0$ reduces to the base case as presented in Section 4.2. This explains why the results displayed in the first row of Tables 3.5a, 3.5b, and 3.5c are the same as the results displayed in the first row of Table 3.2. Second, notice from Table 3.5 that the optimal regular price r^* and the expected effective price E^* under all three policies are increasing in τ. This result can be explained intuitively as follows. As τ increases, (3.19) reveals that the expected spot price (μ^*) will increase (which is also observed in Table 3.5). As the expected spot price (μ^*) increases, the carrier can afford to charge a higher regular price r^* and the expected effective price E^* in period 1 when τ increases. Third, notice from Table 3.5 that the proportion of the shippers who book in advance (q^*) decreases in τ. As τ increases, (3.19) reveals that the expected spot price (μ^*) will increase. As the expected spot price (μ^*) increases, the proportion of the shippers who book in advance (q^*) decreases.

3.7 Conclusion

In this chapter, we have examined a situation in which shippers can either reserve the capacity directly with the carrier at a regular price that is known and fixed in advance or purchase the capacity in the secondary market according to the spot price that is uncertain ex-ante. To entice shippers to buy the capacity directly from the carrier instead of the spot market before demand and spot price uncertainties are resolved, we have analyzed the case when the carrier offers the fractional price matching contract so that there is the shipper receiving a refund based on a "fraction" of the price difference (i.e., the difference between the regular price and the spot price). While we have mainly focused on analyzing the fractional price matching policy throughout the chapter, we have also investigated a different price matching mechanism "partial price matching policy" to show the one-to-one correspondence of these two policies.

By solving a Stackelberg game in which the carrier acts as the leader who sets the regular price and the "fraction" parameter, and the shippers act as followers who decide whether to book directly with the carrier or not, we were able to determine the optimal fractional price matching policy in equilibrium. Specifically, our analytical and numerical results enabled us to draw the following conclusions. First, we have shown that the carrier can use the fractional price matching contract to generate a higher demand from the shippers by increasing the "fractional price matching" in equilibrium. Second, there are multiple optimal regular prices and the optimal "fractions". However, the optimal fractional price matching contract exhibits the following property: if the carrier offers a higher "fraction", then the carrier should increase the regular price to compensate for bearing additional risk. Third, we have found that the optimal fractional price matching contract is "revenue neutral" in the sense that it enables the carrier to obtain the same expected revenue as before (when there was no price matching). This result implies that the carrier can develop a menu of fractional price matching contracts that are "revenue neutral" and let the shipper to choose a specific contract as desired. Finally, we have shown that our results continue to hold when the spot price is dependent on the expected demand in the spot market.

Our study has several limitations that deserve further investigation. First, our model does not capture competition among shippers as the analysis will become highly complicated due to the gaming effects among the shippers. However, it is of interest to extend our model to further investigate how the competition among the shippers leads to different results. Second, in our models, we assume that the spot price is either independent of the demand or it is dependent on the expected demand in the second period. However, one can consider the situation where the spot price depends on the demand as well as the supply (i.e., market capacity). Finally, extending our model to incorporate the issue of dynamic pricing could be of interest also.

Appendix 3.1: Partial Price Matching Policy

In this section, we show that there is a one to one correspondence between the optimal partial price matching policy and the optimal fractional price matching policy, and that the carrier will obtain the same expected optimal revenue under both optimal policies. Under the partial price matching policy, the effective unit price $m(r, \alpha, s)$ is given by:

where $0 \leq \alpha \leq r$. By noting that $m(r, \alpha, s)$ is increasing in α, we can conclude that $\tilde{E} = Exp\{m(r, \alpha, s)\}$ is increasing in α. By using the same argument as presented in Section 3.1, the expected effective price E satisfies (3.2). Also, we can check from (3.3) that the proportion of the shippers who will book with the carrier in period 1 ($q(r, \alpha)$) is decreasing in α. Hence, we have proved the following corollary that is akin to Corollary 1.

Corollary. *Under the partial price matching policy* (r, α)*, the carrier can generate a higher demand (via a higher value of* $q(r, \beta)$*) with a lower expected effective price* $\tilde{E}$ *by reducing the* *matching parameter* α.

Also, when $s \sim U[0,1]$ and $\rho \sim U[0,1]$, it is easy to check from (3.3) that $\mu = \frac{1}{2}$, and

$$\tilde{E} = Exp\{m(r,\alpha,s)\} = \int_0^{r-\alpha} s\,ds + \int_{r-\alpha}^{1} r\,ds \;= r - (\frac{r^2-\alpha^2}{2}),\ \textit{and}$$

$$q(r,\alpha) = 1 - \frac{\tilde{E} - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}. \tag{3.27}$$

While the effective expected price $\tilde{E}$ takes on a different functional form than E given in (3.6) in terms of its price matching parameters (r,α), the proportion of the shippers who will book with the carrier in period 1 $(q(r,\alpha))$ given in (3.27) is identical to (3.7) as a function of $\tilde{E}$ and E, respectively. Hence, we can conclude that there is a one to one correspondence between the proportion of the shippers who will book with the carrier in period 1 between the fractional price matching policy and the partial price matching policy.

For any given (r,α), the expected booking quantity generated by the shippers in period 1 is given by $N \cdot q(r,\alpha)$. Then, the expected revenue to be received by the carrier is equal to: $\Pi(r,\alpha) = E \cdot \min\{N \cdot q(r,\alpha), K\} + \theta\mu \cdot [K - N \cdot q(r,\alpha)]^+$. By noting that $q(r,\alpha)$ given in (3.27) is identical to (3.7) as a function of $\tilde{E}$ and E, it is easy to check that, under the partial price matching policy, the carrier's problem can be formulated as the same problem as defined in (3.8) that has $\tilde{E}$ as a decision variable. Hence, we can use the optimal solution E^* given in (3.12) to retrieve the optimal $\tilde{E^*}$, where $\tilde{E^*} = E^*$. In this case, we have shown that all optimal partial price matching policy $(r^{'},\alpha^{'})$ satisfies: $\tilde{E^*} = r^{'} - \frac{(r^{'})^2-(\alpha^{'})^2}{2}$, where $\tilde{E^*} = E^* = r^* - \beta^* \frac{(r^*)^2}{2}$ as shown in (3.13). Therefore, for any optimal fractional pricing matching policy (r^*,β^*), there is a corresponding optimal partial price matching policy $(r^{'},\alpha^{'})$. More importantly, the expected optimal effective prices under both optimal price matching policies are the same (i.e., $\tilde{E^*} = E^*$). Consequently, we can conclude that the carrier's optimal expected revenue under both optimal policies are the same; i.e., $\Pi(r^{'},\alpha^{'}) = \Pi(r^*,\beta^*)$.

Appendix 3.2: Proofs

Proof of Lemma 1: Suppose not. Then, among all optimal solutions, let us focus on the smallest optimal solution E^* that satisfies $N \cdot (1 - \frac{E^* - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}) > K$. By using the fact that $N > K$, the inequality $N \cdot (1 - \frac{E^* - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}) > K$ implies $E^* < 0.5$. In this case, we can construct a variant of the optimal solution E' that has $E' = E^* + \delta$ with $\delta > 0$, where E' is feasible; i.e., $E' \leq 0.5$; and $N \cdot (1 - \frac{E' - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}) = K$. In this case, it is easy to check from (3.8) that E' will yield a higher revenue for the carrier. This contradicts the supposition that E^* is the smallest optimal solution. This completes our proof.

Proof of Proposition 1: Let us define a decision variable $x := \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}$. By using the fact that $N > K$, constraints (3.10) and (3.11) can be simplified as $(1 - \frac{K}{N}) \leq x \leq 1$. Hence, Program (3.9) can be reformulated as:

$$\begin{aligned} \max_x \quad & N[(1-x)\{(1-\theta)x + \theta\}\tfrac{1}{2} + \{\tfrac{K}{N} - (1-x)\}\tfrac{\theta}{2}] \\ s.t. \quad & (1 - \tfrac{K}{N}) \leq x \leq 1 \end{aligned} \tag{3.28}$$

By considering the first-order condition associated with the objective function (3.28) and by considering the bounds on x, it is easy to check that the optimal solution to problem (3.28) is given by $x^* = \max\{\frac{1}{2}, 1 - \frac{K}{N}\}$. By using the fact that $x = \frac{E - \frac{\theta}{2}}{\frac{1}{2}(1-\theta)}$, we obtain (3.12). Finally, by using the fact that $E = r - \beta\frac{r^2}{2}$ from (3.6), we obtain (3.13).

μ	r_B^*	E_B^*	Π_B^*	r_β^*	β^*	E_β^*	$\Pi_\beta^*,$	r_α^*	α^*	E_α^*	Π_α^*
600	540	540	2.7×10^5	591	0.4471	540	2.7×10^5	580	416	540	2.7×10^5
800	720	720	3.6×10^5	722	0.0278	720	3.6×10^5	750	430	720	3.6×10^5
1000	900	900	4.5×10^5	902	0.0364	900	4.5×10^5	926	424	900	4.5×10^5
1200	1080	1080	5.4×10^5	1083	0.0456	1080	5.4×10^5	1104	410	1080	5.4×10^5
1400	1260	1260	6.3×10^5	1263	0.0516	1260	6.3×10^5	1281	399	1260	6.3×10^5

Table 3.1: Impact of μ: Optimal Pricing Policy

σ	r_B^*	E_B^*	Π_B^*	r_β^*	β^*	E_β^*	Π_β^*	r_α^*	α^*	E_α^*	Π_α^*
300	900	900	4.5×10^5	902.8	0.0364	900	4.5×10^5	926.6	424.2	900	4.5×10^5
400	900	900	4.5×10^5	902.3	0.0204	900	4.5×10^5	964.6	486.7	900	4.5×10^5
500	900	900	4.5×10^5	901.3	0.0087	900	4.5×10^5	1015.9	546.5	900	4.5×10^5
600	900	900	4.5×10^5	900.3	0.0018	900	4.5×10^5	1077.9	603.6	900	4.5×10^5

Table 3.2: Impact of σ: Optimal Policy

μ_ϵ	r_B^*	E_B^*	q_B^*	$\mu_B^*(s)$	Π_B^*
600	1077.3	1077.3	0.0317	1084.1	440320
800	1235.0	1235.0	0.0864	1256.8	522560
1000	1396.5	1396.5	0.1315	1434.3	606460
1200	1561.1	1561.1	0.1866	1615.6	691580
1400	1728.0	1728.0	0.2000	1800.0	777600

μ_ϵ	r_β^*	β^*	E_β^*	q_β^*	$\mu_\beta^*(s)$	Π_β^*
600	1081.2	0.0330	1077.3	0.0317	1084.1	440320
800	1241.4	0.0565	1235.0	0.0864	1256.8	522560
1000	1401.3	0.0459	1396.5	0.1315	1434.3	606460
1200	1626.1	0.5202	1561.1	0.1866	1615.6	691580
1400	1777.7	0.4564	1728.0	0.2000	1800.0	777600

μ_ϵ	r_α^*	α^*	E_α^*	q_α^*	$\mu_\alpha^*(s)$	Π_α^*
600	1108.9	512.2397	1077.3	0.0317	1084.1	440320
800	1261.2	529.0497	1235.0	0.0864	1256.8	522560
1000	1420.0	523.7560	1396.5	0.1315	1434.3	606460
1200	1582.8	512.7746	1561.1	0.1866	1615.6	691580
1400	1748.2	500.8696	1728.0	0.2000	1800	777600

Table 3.3: Impact of μ_ϵ: Optimal Pricing Policy (when $\tau = 1$)

σ_ϵ	r_B^*	E_B^*	q_B^*	$\mu_B^*(s)$	Π_B^*
300	1396.5	1396.5	0.1315	1434.3	606460
400	1396.5	1396.5	0.1315	1434.3	606460
500	1396.5	1396.5	0.1315	1434.3	606460
600	1396.5	1396.5	0.1315	1434.3	606460

σ_ϵ	r_β^*	β^*	E_β^*	q_β^*	$\mu_\beta^*(s)$	Π_β^*
300	1401.3	0.0459	1396.5	0.1315	1434.3	606460
400	1399.7	0.0211	1396.5	0.1315	1434.3	606460
500	1396.8	0.0014	1396.5	0.1315	1434.3	606460
600	1396.5	0.0000	1396.5	0.1315	1434.3	606460

σ_ϵ	r_α^*	α^*	E_α^*	q_α^*	$\mu_\alpha^*(s)$	Π_α^*
300	1420.0	523.7560	1396.5	0.1315	1434.3	606460
400	1459.3	581.1957	1396.5	0.1315	1434.3	606460
500	1509.2	648.8146	1396.5	0.1315	1434.3	606460
600	1568.4	715.8923	1396.5	0.1315	1434.3	606460

Table 3.4: Impact of σ_ϵ: Optimal Policy (when $\tau = 1$)

τ	r_B^*	E_B^*	q_B^*	$\mu_B^*(s)$	Π_B^*
0	900.0	900.0	0.5000	1000.0	450000
0.5	1119.6	1119.6	0.2679	1183.0	519620
1	1396.5	1396.5	0.1315	1434.3	606460
1.5	1694.1	1694.1	0.0512	1711.6	701270

τ	r_β^*	β^*	E_β^*	q_β^*	$\mu_\beta^*(s)$	Π_β^*
0	902.8	0.0364	900.0	0.5000	1000.0	450000
0.5	1153.5	0.3213	1119.6	0.2679	1183.0	519620
1	1401.3	0.0459	1396.5	0.1315	1434.3	606460
1.5	1697.3	0.0287	1694.1	0.0512	1711.6	701270

τ	r_α^*	α^*	E_α^*	q_α^*	$\mu_\alpha^*(s)$	Π_α^*
0	926.6	424.2000	900.0	0.5000	1000.0	450000
0.5	1139.8	512.2507	1119.6	0.2679	1183.0	519620
1	1420.0	523.7560	1396.5	0.1315	1434.3	606460
1.5	1716.9	553.1487	1694.1	0.0512	1711.6	701270

Table 3.5: Impact of τ: Optimal Policy

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